Smith **Math 290** **Fall 2009**

Exam 3

I affirm this work abides by the university's Academic Honesty Policy.

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Print Name, then Sign

Directions:

- **Only write on one side of each page.**
- **Use terminology correctly.**
- **Partial credit is awarded for correct approaches so justify your steps.**

Do any two (2) of these "Computational" problems

C.1. [15 points] The matrix $A = \begin{bmatrix} 3 & 0 & 0 & 0 \\ 4 & 1 & 0 & 0 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 2 \end{bmatrix}$ has characteristic polynomial $p_A(x) = (x-3)(x-1)(x-2)^2$. Determine a basis for the eigenspace $E_A(2)$.

C.2. [15 points] Let $V = \{p \in P_3 \mid p(1) = 0,\ p'(2) = 0\}$. Show that V is a subspace of P_3.

C.3. [15 points] Find a basis for the subspace $V = \{p \in P_3 \mid p(1) = 0,\ p'(2) = 0\}$ of P_3.

Do any two (2) of these "In Class, Text, or Homework" problems

M.1. [20 points] Prove if A is diagonalizable and B is similar to A then B is diagonalizable.

M.2. [20 points] Suppose that A is a square matrix of size n. Prove that the constant term of the characteristic polynomial of A is equal to the determinant of A.

M.3. [20 points] Use a proof by contradiction to show that a single vector can't be an eigenvector of a square matrix A for two different eigenvalues.

M.4. [20 points] Let V be a subspace of M_{nn} and let S be a specific invertible matrix in M_{nn}.Prove that the set $W = \left\{C \in M_{nn} \mid \text{there is a matrix } A \text{ in } V \text{ with } C = S^{-1}AS\right\}$ is also a subspace of M_{nn}.

Do any two (2) of these "Other" problems

T.1. [15 points] Let A be an $n \times n$ matrix and $B = A^2 - 2A + I_n$. show that if $\lambda = 1$ is an eigenvalue of A then B must be singular.

T.2. [15 points] Let $B = S^{-1}AS$ and let $\vec{x}$ be an eigenvector of B corresponding to the eigenvalue λ. Show that $S\vec{x}$ is an eigenvector of A corresponding to λ.

T.3. [15 points] Suppose that A is a 6×6 matrix with exactly two distinct eigenvalues, 2 and -9 and let $E_A(2)$ and $E_A(-9)$ be the corresponding eigenspaces, respectively. Write all possible characteristic polynomials of A that are consistent with $\dim(E_A(2)) = 4$

T.4. [15 points] Let A be an $n \times n$ matrix with eigenvalue λ. If the matrix $A - \lambda I_n$ has rank r, what is the dimension of the eigenspace $E_A(\lambda)$? Explain.