Smith **Math 290** **Fall 2008**

Exam 3 **November** 13, 2008

I affirm this work abides by the university's Academic Honesty Policy.

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Print Name, then Sign

Directions:

- **Only write on one side of each page.**
- **Use terminology correctly.**
- **Partial credit is awarded for correct approaches so justify your steps.**

Do one (1) of these "Computational" problems

C.1. [8, 2 points] Given the matrix $A = \begin{bmatrix} 25 & -8 & 30 \\ 24 & -7 & 30 \\ -12 & 4 & -14 \end{bmatrix}$ and the fact that $P_A(x) = x^3 - 4x^2 + 5x - 2 = (x-2)(x-1)^2$.

1. List the distinct eigenvalues, their algebraic and geometric multiplicities, and their corresponding eigenspaces.
2. Find a diagonal matrix D that is similar to A.

C.2. [10 points] Let $S = \left\{1 + 2x + x^2, 2 + 5x, 3 + 7x + x^2, 1 + x + 3x^2\right\} \subseteq P_2$.Note that S has four elements. Find the dimension of $\langle S \rangle$.

Do this "Subspace" problem

S.1. [15 Points] Let $V = \{p \in P_3 : \ p''(x) - 4p'(x) + xp(x) = 0\}$ .Show that V is a subspace of P_3.

Do this "Proof by Contradiction" problem

PbC.1. [15 Points] Suppose that A is a square matrix. Use a proof by contradiction to show that a single vector may not be an eigenvector of A for two different eigenvalues.

Do any two (2) of these "In Class, Text, or Homework" problems

M.1. [15 Points] Prove that if the matrix A is diagonalizable, then A is similar to A^t.

M.2. [15 Points] Suppose that the set $S = \{\vec{w}_1, \vec{w}_2, \vec{w}_3\}$ is a linearly independent subset of a vector space W. Determine whether or not the set $T = \{2\vec{w}_1 - \vec{w}_3, \vec{w}_1 - 2\vec{w}_2 + 3\vec{w}_3, \vec{w}_2 + \vec{w}_3\}$ is linearly independent.

M.3. [15 Points] Given a linearly independent set $S = \{\vec{v}_1, \vec{v}_2\}$ in a vector space V. Prove that the set $T = \{\alpha\vec{v}_1, \vec{v}_2\}$ is linearly independent if and only if $\alpha \neq 0$.

Do two (2) of these "Other" problems

T.1. [15 Points] Given the subspace $V = \left\{ \vec{v} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \in \mathbf{C}^3 : 2x_1 + 3x_2 - x_3 = 0 \right\}$ of $\mathbf{C}^3$. Find, with proof, a subspace W of $\mathbf{C}^3$ for which $\mathbf{C}^3 = V \oplus W$.

T.2. [15 Points] If A and B are $n \times n$ matrices and A is invertible, show that AB is similar to BA. [This problem is **not** to show that A is similar to B.]

T.3. [15 Points] Suppose A is a square matrix of size n and $S = \{\vec{x}_1, \vec{x}_2, \vec{x}_3, \cdots, \vec{x}_n\}$ is a basis of $\mathbf{C}^n$. Prove that A is nonsingular if and only if $T = \{A\vec{x}_1, A\vec{x}_2, A\vec{x}_3, \cdots, A\vec{x}_n\}$ is a basis of $\mathbf{C}^n$.