Exam 2

I affirm this work abides by the university's Academic Honesty Policy.

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Print Name, then Sign

Directions:

- **Only write on one side of each page.**
- **Use terminology correctly.**
- **Partial credit is awarded for correct approaches so justify your steps.**

Complete the following

D.1. [3 points] Given a finite set of vectors $S \subset \mathbf{C}^m$, define the **span** of S.

D.2. [3 points] Given a matrix A, define the **column space** of A.

D.3. [4 points] If the matrix A is of size $m \times n$. What is the size of the associated matrix L that is a block in the extended reduced row-echelon form of A.

Do and two (2) of these "Computational" problems

C.1. [15 points] We have four ways to compute the column space of a matrix. Use two of them to write the column space of $A = \begin{bmatrix} 1 & 1 & 2 & 0 & 1 \\ 2 & 5 & 4 & 6 & -1 \\ 2 & 3 & 4 & 2 & 1 \end{bmatrix}$ as the span of two different sets **neither of which** consists of vectors that are columns of A.

C.2. [15 points] Let V be the subset of $\mathbf{C}^4$ consisting of all vectors that are orthogonal to both $\begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 2 \\ 0 \\ 1 \\ 1 \end{bmatrix}$. Express V as a span.

C.3. [15 points] Give examples of 3×3 matrices A, B neither of which is the zero matrix O but the product $AB = O$.

Do any two (2) of these "In Class, Text, or Homework" problems

M.1. [15 points] Use block multiplication of partitioned matrices to prove the following portion of Theorem FS (Four Subspaces) about the extended echelon form of a matrix.

1. If A is an $m \times n$ matrix and $N = \left[\begin{array}{c|c} C & K \\ \hline O & L \end{array}\right]$ is the extended reduced row-echelon form for A, prove that the null space of L is contained in the column space of A. More precisely, show that $N(L) \subseteq C(A)$.

M.2. [15 points] Prove property DMAM of matrices. Specifically, prove If $\alpha \in \mathbf{C}$ and $A, B \in M_{mn}$, then $\alpha(A+B) = \alpha A + \alpha B$.

M.3. [15 points] In class we proved that if $A \in M_{mn}$ and $B \in M_{np}$ then the null space of B is contained in the null space of AB, $N(B) \subseteq N(AB)$. Prove that in the special case where A is nonsingular, then $N(B) = N(AB)$.

Do any two (2) of these "Other" problems

T.1. [15 points] Let $S \subseteq \mathbf{C}^n$ be a set containing exactly two vectors. Prove that S is linearly dependent if and only if one of the vectors equals a scalar times the other.

T.2. [15 points] Given a non-singular matrix $A \in M_{nn}$ and a linearly independent set $S = \{\vec{v}_1, \vec{v}_2, \cdots, \vec{v}_p\}$ in $\mathbf{C}^n$, prove that the set $\{A\vec{v}_1, A\vec{v}_2, \cdots, A\vec{v}_n\}$ is also linearly independent.

T.3. [15 points] Prove that if A in an $m \times n$ matrix and B is $n \times p$ then the column space of AB is contained in the column space of A, more precisely, prove that $C(AB) \subseteq C(A)$.