Exam 1

I affirm this work abides by the university's Academic Honesty Policy.

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Print Name, then Sign

Directions:

- **Only write on one side of each page.**
- **Use terminology correctly.**
- **Partial credit is awarded for correct approaches so justify your steps.**

Complete the following definitions

D.1. [3 points] Given a finite set of vectors $S \subset \mathbf{C}^m$, the **span** of S is

D.2. [3 points] The **null space** of a matrix A, denoted $N(A)$ is

D.3. [4 points] A matrix A is in **reduced row-echelon form** if it meets all of the following conditions

Do both of these "Computational" problems

C.1. [10, 5 points] Solve the following system of linear equations by hand. Write the solution set using column vector notation.

$$\begin{aligned} x+3y+4w &= 3 \\ -z-5w &= -2 \\ x+3y+z+9w &= 5 \\ 2x+6y+8w &= 6 \end{aligned}$$

C.2. [15 points] Suppose we run the sequence of three row operations: $\left[A \mid \vec{b}\right] \xrightarrow{R_2 \leftrightarrow R_4} [B \mid \vec{c}] \xrightarrow{2R_3} \left[C \mid \vec{d}\right]$ where $[D \mid \vec{e}] = \begin{bmatrix} 1 & 2 & -1 \\ 3 & 4 & 0 \\ 1 & -1 & 2 \\ 0 & 1 & 1 \end{bmatrix}$. What is C if C is the result of running the row operation $-4R_2+R_1$ on A?

Do any two (2) of these "In Class, Text, or Homework" problems

M.1. [15 points] Is there a parabola whose graph passes through the points $(1,3)$, $(2,6)$, $(4,-1)$, $(5,0)$? Give careful reasons for your answer.

M.2. [15 points] Property **SMAC** of column vectors is: if $\alpha, \beta \in \mathbf{C}$ and $\mathbf{u} \in \mathbf{C}^m$, then $\alpha(\beta\mathbf{u}) = (\alpha\beta)\mathbf{u}$. Prove this property and write your proof in the style of the proof of Property **DSAC** given in the textbook.

M.3. [15 points] Prove that a system of linear equations is homogeneous **if and only if** it has the zero vector as a solution.

Do any two (2) of these "Other" problems

T.1. [10, 5 points] Suppose that α is any constant and that $\mathbf{u} = \begin{bmatrix} u_1 \\ \vdots \\ u_n \end{bmatrix}$ is a solution of the homogeneous system of linear equations $LS(A, \mathbf{0})$. Prove that $\mathbf{t} = \begin{bmatrix} \alpha u_1 \\ \vdots \\ \alpha u_n \end{bmatrix}$ is also a solution of $LS(A, \mathbf{0})$. [Be sure to explicitly show that $\mathbf{t}$ solves the system of equations.]

T.2. [15 points] Consider the system of m linear equations in n variables $LS(A, \mathbf{b})$, and suppose that the vector $\vec{b}$ equals twice one column vector of A plus five times a different column vector of A. More precisely, there are two column indices j_1 and j_2 such that for each i, $1 \leq i \leq m$, it is the case that $[\mathbf{b}]_i = 2[A]_{i,j_1} + 5[A]_{i,j_2}$. Prove that the system is consistent.

T.3. [15 points] Let $S = \{\vec{w}_1, \vec{w}_2, \vec{w}_3\}$ and $T = \{\vec{w}_1, \vec{w}_2, \vec{w}_3, \vec{w}_1 + 2\vec{w}_2 + 3\vec{w}_3\}$ be subsets of $\mathbf{C}^m$. Prove that $\langle S \rangle = \langle T \rangle$.