Exam 1

I affirm this work abides by the university's Academic Honesty Policy.

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Print Name, then Sign

Directions:

- **Only write on one side of each page.**
- **Use terminology correctly.**
- **Partial credit is awarded for correct approaches so justify your steps.**

Define all of the following

D.1. [5 points] A **consistent** system of linear equations.

D.2. [5 points] The **null space** of a matrix.

D.3. [5 points] The $n \times n$ **identity matrix.**

Do both of these "Computational" problems

C.1. [15 points] Solve the following system of linear equations by hand. Write the solution set using column vector notation.

$$\begin{aligned} x_1 + 2x_2 - x_3 + x_4 &= 1 \\ x_2 + x_3 - x_4 &= 3 \\ -x_1 + x_2 + 7x_3 - x_4 &= 5 \end{aligned}$$

C.2. [15 points] **Without solving** one can see that the number of solutions of the linear system

$$\begin{aligned} x_1 - 5x_2 + 3x_3 - x_4 &= 7 \\ 3x_1 + x_2 - 4x_3 &= 2 \\ x_2 + x_4 &= 5 \end{aligned}$$

1. is definitely either 2. or 3. below. No other possibilities exist, but to determine which is the case would require solving the system.
2. is zero
3. is infinite
4. is definitely not either 2. or 3.
5. might be 2. or 3. but other possibilities exist.

Briefly explain your answer.

Do any two (2) of these "In Class, Text, or Homework" problems

M.1. [15 points] Find a 4×5 matrix A, that is **not** in reduced row-echelon form, whose null space is the set

$$S = \left\{ \begin{bmatrix} 5x_2 - 6x_4 \\ x_2 \\ 2x_2 - 5x_4 \\ x_4 \\ x_4 \end{bmatrix} : x_2,\ x_4 \in \mathbf{C} \right\}$$

M.2. [15 points] Suppose that A is the coefficient matrix of a consistent linear system of equations and that two of the columns of A are identical. Prove that there must be an infinite number of solutions to the system of equations $LS\left(A, \vec{0}\right)$.

M.3. [15 points] Suppose we run the sequence of three row operations: $\left[A \mid \vec{b}\right] \xrightarrow{R_2 \leftrightarrow R_4} [B \mid \vec{c}] \xrightarrow{\quad 3R_3 \quad} \left[C \mid \vec{d}\right]$ where

$$[D \mid \vec{e}] = \begin{bmatrix} 1 & 2 & -1 \\ 3 & 4 & 0 \\ 1 & -1 & 2 \\ 0 & 1 & 1 \end{bmatrix}$$

What is $\vec{b}$?

Do any one (1) of these "Other" problems

T.1. [15 points] Suppose A is a square coefficient matrix for a consistent system of linear equations $LS\left(A, \vec{b}\right)$ in which one row of A, say the jth row, is double the ith row. Prove that the solution set for the linear system $LS\left(A, \vec{b}\right)$ is infinite.

T.2. [15 points] Prove that if the reduced row-echelon form of $\left[A \mid \vec{b}\right]$ is $[B \mid \vec{c}]$ then the reduced row-echelon form of A is B.

T.3. [15 points] Use the principle of mathematical induction to prove that if $LS\left(A_n, \vec{b}_n\right)$ is the linear system of equations resulting from a sequence of n equation operations on the linear system $LS\left(A_0, \vec{b}_0\right)$ then the two systems have the same solution sets. You may use (but don't prove) Professor Beezer's Theorem EOPSS (Equation Operations Preserve Solution Sets) which states that the result of any **single** equation operation yields a system of equations with the same solution set.