Mathematics 290-A Exam 1 Fall 2006

September 12, 2006

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Name

Technology used: ______________________________

Directions:

- **Only write on one side of each page.**
- **Use terminology correctly.**
- **Partial credit is awarded for correct approaches so justify your steps.**

Do any six (6) of the following problems.

1. Solve the following system of linear equations by hand. Write the solution set using column vector notation.

$$\begin{aligned} x_1 + 3x_2 + x_3 + 5x_4 &= 6 \\ 2x_1 + x_2 - 3x_3 &= 2 \\ x_2 + x_3 + 2x_4 &= 2 \\ -x_1 + 2x_2 + 4x_3 + 4x_4 &= 3 \end{aligned}$$

2. Find values of the coefficients for polynomial $f(x) = ax^3 + bx^2 + cx + d$ such that $f(1) = -3$, $f(2) = 8$, $f'(1) = 0$, and $f'(2) = 27$.

3. Prove the following part of Theorem EOPSS (Equation Operations Preserve Solution Sets); Given linear system $LS(A, \mathbf{b})$ with solution set S, and linear system $LS(B, \mathbf{c})$ with solution set T that results from the following row operation on the augmented matrices (where $\alpha \neq 0$): $[A|\mathbf{b}] \xrightarrow{\alpha R_i} [B|\mathbf{c}]$. Prove that $S = T$.

4. Suppose A, B, and C are $m \times n$ matrices. Explain why if A is row-equivalent to B and B is row-equivalent to C then A is row-equivalent to C.

5. Prove Theorem NSRRI: Suppose that A is a square matrix and B is a row-equivalent matrix in reduced row-echelon form. Then A is nonsingular if and only if B is the identity matrix.

6. A zoo charges \$6 for adults, \$3 for students, and \$0.50 for children. One morning 79 people enter and pay a total of \$207. Determine the possible numbers of adults, students, and children.

7. Suppose that $\mathbf{u} = \begin{bmatrix} u_1 \\ \vdots \\ u_n \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} v_1 \\ \vdots \\ v_n \end{bmatrix}$ are solutions of the homogeneous system of linear equations $LS(A, \mathbf{0})$. Prove that $\mathbf{t} = \begin{bmatrix} u_1 + v_1 \\ \vdots \\ u_n + v_n \end{bmatrix}$ is also a solution of $LS(A, \mathbf{0})$. [Be sure to explicitly show that $\mathbf{t}$ solves the system of equations.]