Mathematics 280 **Spring 2008**

Exam 3

March 27, 2008

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Technology used: ____

Only write on one side of each page.

- Show all of your work. Calculators may be used for numerical calculations and answer checking only.

Do six (6) of the following problems

1. (8, 7 points) Given the function $f(x, y, z) = \ln\left(\frac{x^2}{25} + \frac{y^2}{16} - \frac{z}{9}\right)$

   (a) Find an equation for the level surface of f that passes through the point $\left(5\sqrt{2}, 4, 9\right)$.

   (b) Draw a reasonably careful sketch of that level surface.

2. (3, 12 points) Suppose $f(x, y) = \frac{9x^2 - y^2}{x^2 + 4y^2}$ for all $(x, y) \neq (0, 0)$.

   (a) Is there a number k that makes the function g given below continuous at $(0, 0)$?

$$g(x, y) = \begin{cases} f(x, y) & \text{if } (x, y) \neq (0, 0) \\ k & \text{if } (x, y) = (0, 0) \end{cases}$$

   (b) Why or why not?

3. Reparamatrize the following curve by arclength. That is, express this curve in terms of the arclength parameter $s(t) = \int_0^t \|\vec{r}'(\tau)\| \, d\tau$.

$$\vec{r}(t) = (\cos t + t \sin(t))\, \widehat{\mathbf{i}} + (\sin t - t \cos t)\, \widehat{\mathbf{j}}, \quad \frac{\pi}{2} \leq t \leq \pi$$

4. (5, 5, 5 points) Find $\mathbf{T}, \mathbf{N}$ and the curvature κ for the parametrized space curve $\vec{r}(t) = (\cos^3 t)\, \widehat{\mathbf{i}} + \left(\sin^3 t\right) \widehat{\mathbf{j}}$, $0 < t < \pi/2$.

5. If $w = \sin(2x + y)$, $x = \sin(\pi s)$, and $y = rs$, find the value of the following second partial derivative at the point where $r = \pi$ and $s = 1$.

$$\frac{\partial^2 w}{\partial r \partial s}$$

6. Write a chain rule formula for $\frac{\partial w}{\partial t}$ if $w = f(x, y)$, $x = g(t, s)$, and $y = h(t, s, x)$ are all differentiable functions of their respective input variables.

7. The space curve $\vec{r}\,(t) = \left(\frac{1}{3}t^3\right)\hat{\mathbf{i}} + (t^2)\,\hat{\mathbf{j}} + (2t)\,\hat{\mathbf{k}}$ has unit tangent vector $\mathbf{T}\,(t) = \frac{t^2}{t^2+2}\hat{\mathbf{i}} + \frac{2t}{t^2+2}\hat{\mathbf{j}} + \frac{2}{t^2+2}\hat{\mathbf{k}}$ and taking the derivative we find that

$$\frac{d\mathbf{T}}{dt} = \left(\frac{4t}{(t^2+2)^2}\right)\hat{\mathbf{i}} + \left(\frac{-2t^2+4}{(t^2+2)^2}\right)\hat{\mathbf{j}} + \left(\frac{-4t}{(t^2+2)^2}\right)\hat{\mathbf{k}}$$

   (a) What is an equation of the osculating plane for this curve at the point $x = 2$?

   (b) What are the center and radius of the osculating circle (also known as the circle of curvature) for this curve at the point where $t = 2$?

8. We showed in class that if $\vec{r}\,(t) = \langle x\,(t)\,, y\,(t)\,, z\,(t)\rangle$ is a smooth parametrized space curve satisfying $\|\vec{r}\,(t)\| = c$ for every t in the domain of $\vec{r}$ then $\vec{r}\,(t)$ and $\vec{r}'\,(t)$ are perpendicular vectors for every t in that domain. By using components and integration, show that the converse is also true by proving the following.

   Theorem If $\vec{r}\,(t) = \langle x\,(t)\,, y\,(t)\,, z\,(t)\rangle$ is a smooth parametrized space curve in which $\vec{r}\,(t) \cdot \vec{r}'\,(t) = 0$ for every t in the domain of $\vec{r}$, then there is a constant c for which $\|\vec{r}\,(t)\| = c$ for every t in the domain of $\vec{r}$.