Mathematics $280A, 280B$ **Fall 2012**

Exam 3

November 13, 2012

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Name

Technology used: ____ **Only write on one side of each page.**
Show all of your work.
Calculators may be used for numerical calculations and answer checking only.

You MUST do A. and one part of B.

A. [10 points] Reverse the order of integration in the following double integral **Do Not** evaluate the integral.

$$\int_0^3 \int_{\sqrt{x/3}}^1 e^{\left(y^3\right)}\, dy\, dx.$$

B. Do **one** (1) of the following:

(a) [7, 8 points] Find and classify all local maxima, local minima and saddle points of the function $f(x, y) = x^4 + y^4 + 4xy$.

(b) [15 points] Find the absolute minimum value of the function $f(x, y) = 48xy - 32x^3 - 24y^2$ on the rectangular plate $0 \leq x \leq 1$, $0 \leq y \leq 1$.

Do any FIVE (5) of the following

1. [10, 5 points] Given $f(x, y) = 49 - x^2 - y^2$

   (a) If possible, maximize $f(x, y)$ subject to the constraint $x + 3y = 10$.

   (b) Explain why or why not this constrained optimization has an absolute maximum.

2. [15 points] The area charge density function for a region in the xy-plane bounded by the cardioid $r = 1 + \sin(\theta)$ is proportional to the square of the distance from the origin with the maximum σ_0 occuring at the point with polar coordinates $[2, \pi/2]$ .Express the total charge as an iterated double integral in polar coordinates. **Do Not evaluate** your integral.

3. [15 points] It can be shown that the improper integral $I = \int_0^\infty e^{-x^2}\, dx$ converges. The usual way to determine the value is to first calculate its square

$$\begin{aligned} I^2 &= \left(\int_0^\infty e^{-x^2}\, dx\right)\left(\int_0^\infty e^{-y^2}\, dy\right) \\ &= \int_0^\infty \int_0^\infty e^{-\left(x^2+y^2\right)}\, dx dy. \end{aligned}$$

   Evaluate the last integral using polar coordinates and solve the resulting equation for I.

4. [15 points] Change the order of integration to the order $dz\, dx\, dy$, but **do not evaluate**, the following triple integral.

$$\int_0^2 \int_0^{4-x^2} \int_0^x \frac{\sin(2z)}{4-z}\, dy\, dz\, dx.$$

5. [15 points] Each point of the portion of the solid sphere $\rho \leq a$ that lies between the cone $\phi = \frac{\pi}{3}$ and the plane $z = 0$ has a volume charge density proportional to the distance of the point from the origin. The maximum volume charge density of δ_0 occurs along the circle where the cone meets the sphere. Find the total charge on the solid.

6. [15 points] Let n be a positive integer. Set up and evaluate a definite integral that gives the length of a helix that wraps 17 times around the lateral side of a right circular cylinder of radius R and height H with a constant pitch (so each wrap rises the same distance up the cylinder). Your answer should not have any integral signs and will involve the letters R and H.