Mathematics $280A, 280B$ **Fall 2012**

Exam 2

October 11, 2012

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Name

Technology used: ____

Only write on one side of each page.
Show all of your work.
Calculators may be used for numerical calculations and answer checking only.

1. [10 points] Simplify the following vector $\mathbf{v}$ and express it using the unit vectors $\mathbf{i}, \mathbf{j}, \mathbf{k}$.

$$\mathbf{v} = (\langle 1, 3, -1\rangle \cdot \langle -1, 0, -2\rangle)\langle 2, 1, 4\rangle + (5\langle -3, -2, -1\rangle - 3\langle 1, 2, 3\rangle)$$

2. [15 points] Draw a tree diagram and write a Chain Rule formula for the derivative $\frac{\partial w}{\partial t}$ where

$$w = g(x, y, t), \quad x = h(u, v, t), \quad y = f(v, t)$$

3. [8, 7 points] Do both of the following

   (a) Write an equation for the plane that is tangent to the surface $z = e^{-\left(x^2+y^2\right)}$ at the point $(0, 0, 1)$.

   (b) A certain plane passes through the point $P_0(2, 3, -1)$ and the projection of the vector $\overrightarrow{OP}$ onto the normal vector of the plane is $\mathbf{i}+5\mathbf{j}-2\mathbf{k}$.Write an equation in standard form of the plane.

4. [8, 7 points] Do both of the following.

   (a) Articulate how gradient vectors are related to level curves/surfaces and greatest rate of change.

   (b) Compute the directional derivative of the function $z = f(x, y) = \cos(xy^2)$ at the point $(0, 0, 1)$ and in the direction of the vector $\mathbf{v} = 4\mathbf{i} - 3\mathbf{j}$.

5. [9, 6 points] Do both of the following

   (a) Around the point $(1, 0)$, is $f(x, y) = x^2(y + 1)$ more sensitive to changes in x or to changes in y? Why?

   (b) What ratio of dx to dy will make df equal 0 at $(1, 0)$?

6. [15 points] Do **one** (1) of the following

   (a) Suppose $\vec{r}(t)$ is a vector-valued function with the property that for every t in the domain of $\vec{r}$, $\|\vec{r}(t)\| = 4$.
   Express $\|\vec{r}(t)\|^2$ as a dot product and take the derivative of the result to show that for every t in the domain, $\vec{r}(t)$ is orthogonal to it's derivative $\vec{r}'(t)$.

   (b) If $\mathbf{u}_1$ and $\mathbf{u}_2$ are orthogonal unit vectors and $\mathbf{v} = a\mathbf{u}_1 + b\mathbf{u}_2$, show that $\mathbf{v} = (\mathbf{v} \cdot \mathbf{u}_1)\mathbf{u}_1 + (\mathbf{v} \cdot \mathbf{u}_2)\mathbf{u}_2$.

7. [8, 7 points] If $\mathbf{a} = \langle 1, 4\rangle$ and $\mathbf{b} = \langle 2, -3\rangle$

   (a) Compute the vector projection, $\mathbf{c} = \text{proj}_{\mathbf{b}}\mathbf{a}$,of $\mathbf{a}$ onto $\mathbf{b}$.

   (b) Show that $\mathbf{b}$ is orthogonal to $\mathbf{a} - \mathbf{c}$ by using the dot product.