Mathematics 232-B — Spring 2000

Exam 3

April 21, 2000

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Name

Technology used: ____

Textbook/Notes used: ____

Directions: Be sure to include in-line citations, including page numbers if appropriate, every time you use a text or notes or technology. Include a careful sketch of any graph obtained by technology in solving a problem. **Only write on one side of each page.**

The Problems

1. Show that the function $T : R^2 \to R^3$ is a linear transformation.

$$T\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} x_1 + 5x_2 \\ 0 \\ 2x_1 - 3x_2 \end{bmatrix}.$$

2. Do **one** of the following.

   (a) Without using technology, compute the determinant of the matrix

$$\begin{bmatrix} 0 & -1 & 0 & 1 \\ -2 & 3 & 1 & 4 \\ 1 & -2 & 2 & 3 \\ 0 & 1 & 0 & -2 \end{bmatrix}.$$

   (b) The characteristic polynomial of $\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ is $\lambda^2 (\lambda - 1)^2$. Find the eigenvalues and determine a basis for each eigenspace.

3. Do **one** of the following.

   (a) Suppose $\overrightarrow{v}$ is an eigenvector of the matrix A with associated eigenvalue 3. Explain why $\overrightarrow{v}$ is also an eigenvector for the matrix $A^2 + 4I_n$. What is the associated eigenvalue?

   (b) Suppose that A is a 4×4 matrix with exactly two distinct eigenvalues, 5 and -9 and let E_5 and E_{-9} be the corresponding eigenspaces, respectively. Write all possible characteristic polynomials of A, in factored form, that are consistent with $\dim(E_5) = 1$.

4. Do **one** of the following.

   (a) Is the matrix $A = \begin{bmatrix} 1 & 0 \\ 10 & 2 \end{bmatrix}$ diagonalizable? If not, explain why not. If so, find an invertible matrix S for which $S^{-1}AS$ is diagonal.

(b) The matrices $A = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix}$ are similar. Exhibit a matrix S for which $B = S^{-1}AS$.

5. Do **two** of the following.

(a) Show that the set, $W = \left\{ A \in R^{3\times 3} : \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \text{ is an eigenvector of } A \right\}$ is a subspace of $R^{3\times 3}$.

(b) Find a basis for the subspace $W = \{A \in R^{2\times 2} : \operatorname{trace}(A) = 0\}$. Be sure to show that your basis both spans W and is linearly independent.

(c) Suppose $T : R^2 \to R^3$ is a linear transformation such that $T\begin{bmatrix} 3 \\ -5 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$ and $T\begin{bmatrix} -1 \\ 2 \end{bmatrix} = \begin{bmatrix} 3 \\ 0 \\ -2 \end{bmatrix}$. Determine $T\begin{bmatrix} 7 \\ -11 \end{bmatrix}$.