Mathematics 232-B Spring 2000

Exam 1

February 14, 2000

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Name

Technology used: __________________________

Textbook/Notes used: __________________________

Directions: Be sure to include in-line citations, including page numbers if appropriate, every time you use a text or notes or technology. Include a careful sketch of any graph obtained by technology in solving a problem. **Only write on one side of each page.**

You are to turn in **five** problems. Additional problems from number 5 may be substituted for any of the other problems.

The Problems

1. Do **one** of the following **by hand.**

   (a) Determine if the matrix A below is invertible and, if it is, find its inverse.

$$A = \begin{bmatrix} 1 & 1 & 2 \\ 2 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix}$$

   (b) Use Gauss-Jordan elimination to solve the following system of equations.

$$\begin{aligned} x_1 + 2x_2 - x_3 + 3x_4 &= 2 \\ 2x_1 + 4x_2 - x_3 + 6x_4 &= 5 \\ x_2 \qquad + 2x_4 &= 3 \end{aligned}$$

2. Do **one** of the following.

   (a) Find a polynomial $f(t)$ of degree 3 such that $f(1) = 1$, $f(2) = 5$, $f'(1) = 2$, and $f'(2) = 9$.

   (b) The matrix $A = \begin{bmatrix} 4 & -2 \\ -1 & 3 \end{bmatrix}$ has the property that there is at least one vector $\overrightarrow{x}$ for which $A\overrightarrow{x} = 5\overrightarrow{x}$. Find all such vectors.

3. Label the following statements as being true or false.

   (a) The rank of a matrix is equal to the number of its nonzero columns.

   (b) The rank of a matrix is equal to the number of its nonzero rows.

   (c) The $m \times n$ zero matrix is the only $m \times n$ matrix having rank 0.

   (d) Elementary row operations preserve rank.

   (e) An $n \times n$ matrix of rank n is invertible.

   (f) It is possible for a 3×5 matrix to have rank 4.

(g) It is possible for a 5×3 matrix to have rank 4.

4. Given an $(m \times n)$ matrix A and the n standard basis vectors

$$\overrightarrow{e_1} = \begin{bmatrix} 1 \\ \vdots \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \cdots, .\overrightarrow{e_n} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ \vdots \\ 1 \end{bmatrix}. \text{ Show that } A = \begin{bmatrix} | & | & & | \\ A\overrightarrow{e_1} & A\overrightarrow{e_2} & \cdots & A\overrightarrow{e_n} \\ | & | & & | \end{bmatrix}.$$

5. Do **one** of the following.

(a) Problem 2 of the quiz can be used to prove the fact that invertible matrices are exactly those matrices that can be written as products of elementary matrices. Use the fact (which you do not need to prove) that all elementary matrices are invertible to partially justify this claim by proving the following lemma.

Lemma 1 *If $E_1, \cdots, E_k$ are elementary matrices for which $(E_k \cdots E_2 E_1)\, A = rref(A)$ and A is invertible then A can be written as a product of elementary matrices.*

(b) Suppose $\overrightarrow{v}_1, \overrightarrow{v}_2, \overrightarrow{v}_3$ is a linearly independent set in R^5. Is the set of vectors $2\overrightarrow{v}_1 + \overrightarrow{v}_2 + 3\overrightarrow{v}_3$, $\overrightarrow{v}_2 + 5\overrightarrow{v}_3$, $3\overrightarrow{v}_1 + \overrightarrow{v}_2 + 2\overrightarrow{v}_3$ linearly dependent or independent?

(c) Suppose $A_{n\times m}$ and $B_{m\times n}$ are matrices such that $AB = I_n$. Let $\overrightarrow{b}$ be a particular vector in R^n. Show that the system of equations $A\overrightarrow{x} = \overrightarrow{b}$ must be consistent.