Mathematics 232-B

Spring 1999

Exam 1

September 24

_____
Name

Technology used: _____

Textbook/Notes used: _____

Directions: Be sure to include in-line citations, including page numbers if appropriate, every time you use a text or notes or technology. Include a careful sketch of any graph obtained by technology in solving a problem. **Only write on one side of each page.**

The Problems

You may use technology for any problem other than the first.

1. Solve the following system of equations **by hand**.

$$\left| \begin{array}{c} x_1 - x_2 - 2x_3 - x_4 = -3 \\ 3x_1 - 3x_2 - 2x_3 + 5x_4 = 7 \\ 2x_1 - 2x_2 - 3x_3 \quad = -2 \end{array} \right|$$

2. Do one of the following

   (a) Find the inverse of the matrix below or show that the inverse does not exist.

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 3 & 8 \\ 1 & 2 & 2 \end{bmatrix}$$

   (b) Determine if the following collection of vectors in $\mathbf{R}^4$ are linearly independent or dependent.

$$\overrightarrow{v_1} = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}, \ \overrightarrow{v_2} = \begin{bmatrix} 5 \\ 6 \\ 7 \\ 8 \end{bmatrix}, \ \overrightarrow{v_3} = \begin{bmatrix} 9 \\ 10 \\ 11 \\ 12 \end{bmatrix}$$

3. Do one of the following

   (a) Find all vectors in $\mathbf{R}^4$ whose dot product with each of the following vectors is 0. That is, find all $\overrightarrow{x}$ such that $\overrightarrow{x} \cdot \overrightarrow{v_i} = 0$ for $i = 1, 2, 3$.

$$\overrightarrow{v_1} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \ \overrightarrow{v_2} = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}, \ \overrightarrow{v_3} = \begin{bmatrix} 1 \\ 9 \\ 9 \\ 7 \end{bmatrix}$$

   (b) Find a polynomial of degree 3 whose graph goes through the points $(2, -1)$, $(3, -59)$, $(-1, 5)$, and $(-2, -29)$.

4. Do one of the following

   (a) Suppose we know that a (2×2) invertible matrix A has all entries integers and that all the entries in A^{-1} are also integers. Show that the only possible values for the determinant of A are 1 and -1.

   (b) Is is possible to have an invertible (3×3) matrix A with $AA = O$? (Here O represents the (3×3) zero matrix.)

5. Do one of the following

   (a) Give an example of a (2×3) matrix A and a (3×2) matrix B for which $AB = I_2$.

   (b) Suppose A is a (3×3) matrix. Show it is always possible to find a non-zero (3×3) matrix B with $AB = O$ where O represents the (3×3) zero matrix. [Hint: consider the solutions of the system of equations $B\overrightarrow{x} = \overrightarrow{\theta}$.