Mathematics 221 Fall 2003

Exam 1

September 23

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Name

Directions: Cite any use of technology. For partial credit, if you are unsure of your answer to a problem be sure to describe what you do know and where you think your error is. Include a careful sketch of any graph obtained by technology in solving a problem. **Only write on one side of each page.**

The Problems

I (10 points each) Do any three (3) of the following problems.

1. Write inequalities that describe the region consisting of all points between, but not on, the spheres of radius r and R centered at the origin, where $r < R$.
2. Find the area of the triangle formed by points $P\,(2, 0, -3)$, $Q\,(3, 1, 0)$, $R\,(5, 2, 2)$.
3. If $\overrightarrow{\mathbf{a}}$, $\overrightarrow{\mathbf{b}}$,and $\overrightarrow{\mathbf{c}}$ are vectors in $\mathbf{R}^3$, state whether each expression is meaningful. If it is, state whether it is a scalar or a vector.
   (a) $\overrightarrow{\mathbf{a}} \cdot \left(\overrightarrow{\mathbf{b}} \times \overrightarrow{\mathbf{c}}\right)$
   (b) $\overrightarrow{\mathbf{a}} \times \left(\overrightarrow{\mathbf{b}} \cdot \overrightarrow{\mathbf{c}}\right)$
   (c) $\overrightarrow{\mathbf{a}} \times \left(\overrightarrow{\mathbf{b}} \times \overrightarrow{\mathbf{c}}\right)$
   (d) $\left(\overrightarrow{\mathbf{a}} \cdot \overrightarrow{\mathbf{b}}\right) \times \overrightarrow{\mathbf{c}}$
   (e) $\left(\overrightarrow{\mathbf{a}} \cdot \overrightarrow{\mathbf{b}}\right) \times \left(\overrightarrow{\mathbf{c}} \cdot \overrightarrow{\mathbf{d}}\right)$
   (f) $\left(\overrightarrow{\mathbf{a}} \times \overrightarrow{\mathbf{b}}\right) \cdot \left(\overrightarrow{\mathbf{c}} \times \overrightarrow{\mathbf{d}}\right)$
4. Identify the following quadric surfaces by name (e.g., sphere, hyperboloid of one sheet, et cetera). **Do Not Sketch.**
   (a) $y^2 + z^2 = 1 - 4x^2$
   (b) $y^2 + z^2 = x$
   (c) $y^2 + z^2 = 1$
   (d) $y = z^2 - x^2$
   (e) $y^2 + z^2 = 1 + x^2$
   (f) $4x^2 - y^2 + z^2 + 8x + 8z = -20$

II (15 points each) Do any two (2) of the following problems.

1. Suppose $\overrightarrow{a}$ is a three-dimensional unit vector in the first octant that starts at the origin and makes angles of $60°$ and $72°$ with the positive x- and y- axes, respectively. What are the components of $\overrightarrow{a}$?
2. Given non-zero vectors $\overrightarrow{a}$, $\overrightarrow{b}$ for which $\mathrm{Proj}_{\overrightarrow{a}}\overrightarrow{b}$ is also non-zero, show that the vector $\overrightarrow{b} - \mathrm{Proj}_{\overrightarrow{a}}\overrightarrow{b}$ is orthogonal to $\overrightarrow{a}$. Do not give a geometric argument: use the dot product.
3. Suppose L is the line that passes through the point $P\,(0, 2, -1)$ and is parallel to the line with parametric equations $x = 1 + 2t$, $y = 3t$, $z = 5 - 7t$. Find the points where L meets the three coordinate planes.

4. Write an equation in parametric form for the line of intersection of the planes $x+y+z=1$ and $x+z=0$.

III (20 points each) Do any two (2) of the following problems.

1. Write an equation for either of the planes that are parallel to the plane $x+2y-2z=1$ and are two units away from it.
2. If $\overrightarrow{a}\cdot\overrightarrow{c}=\overrightarrow{b}\cdot\overrightarrow{c}$, must it be the case that $\overrightarrow{a}=\overrightarrow{b}$? If so, explain why. If not, provide a counterexample.
3. Find an equation for the plane that contains the parallel lines
$$\frac{x-3}{2}=\frac{y+4}{5}=\frac{3-z}{6}\ \text{and}\ \frac{x+4}{2}=\frac{y-7}{5}=\frac{z+1}{6}.$$
4. Find parametric equations for the line through the point $(0,1,2)$ that is parallel to the plane $x+y+z=2$ and perpendicular to the line $x=1+t,\ y=1-t,\ z=2t$.