Mathematics $122 - A$ **Fall 2002**

Final Examination

December 19, 2002

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Name

Technology used:______________________________ Directions:

Be sure to include in-line citations, including page numbers if appropriate, every time you use a text or notes or technology. Include a careful sketch of any graph obtained by technology in solving a problem. **Only write on one side of each page.**

The Problems

1. $(2, 2, 3, 3$ points)

   (a) Give an example of a convergent infinite series.

   (b) give an example of a divergent improper integral.

   (c) Give an example of a monotone, non-decreasing, unbounded sequence

   (d) Briefly explain what it means for an infinite series $\sum_{k=1}^{\infty} a(k)$ to converge. Your answer must include a limit to be correct.

2. (20 points each) Without using your calculator, evaluate three (3) of the following integrals.

   (a)
   $$\int \frac{\sin(x)}{\sqrt{1+\cos(x)}}\ dx$$

   (b)
   $$\int \frac{4x+4}{x(x^2+1)}\ dx$$

   (c)
   $$\int_{-1}^{\infty} \frac{1}{x^{4/3}}\ dx$$

   (d)
   $$\int \sqrt{\frac{1+x}{1-x}}\ dx$$

3. (15 points) Do **one** (1) of the following.

   (a) A solid object has a base that lies in the region in the first quadrant between $y = x^3$, $x = 1$ and the x-axis. Cross sections of the solid perpendicular to the y-axis are squares. Use the method of cross-sectional areas to find the volume of the solid.

   (b) Find the volume of the solid obtained by rotating the region bounded by $y = 2x - x^2$ and the x-axis about the line $x = -1$. Specify whether you are using the method of cylindrical shells or the washer method.

4. (15 points) In chapter 5 we showed that the volume of a solid is completely determined by the areas of its cross sections sliced perpendicular to an axis along its length. We translated this fact into the language of calculus obtaining $V = \int_a^b A(x)\ dx$. Outline the complete Riemann sum development of this formula. Start with the fact that the solid lies between a and b on the x - axis.

5. (15 points) Set up the definite integral that models the following. **Do not evaluate the integral.**

   A tank in the shape of a (point down) cone is 12 feet high and has a circular base of radius 4 feet. If the cone is filled to a depth of 5 feet with a syrup weighing 100 pounds per cubic foot, what is the work done to pump the syrup out of a spout 3 feet above the top of the cone? [Hint: use similar triangles.]

6. (15 points) Do **one** (1) of the following.

   (a) Use the error bound for the Trapezoid Rule to determine a value of n that guarantees the trapezoidal estimate for the following integral is accurate to within 0.001. [You will probably need your calculator.]

$$\int_{0.5}^{2} \sin(x^3)\ dx$$

   (b) The formula for computing a Taylor Series for a function f at $x = c$ is

$$\sum_{k\,=\,0}^{\infty} \frac{f^{(k)}(c)}{k!}(x-c)^k$$

   Use this formula to compute the Taylor Series with $c = 1$ for the function $f(x) = (3 + 2x)^{-1}$. Write your answer in Sigma notation. [Hint: be sure to use the chain rule!. Also, to see the pattern, keep track of all multiples of 2 that occur in your derivatives.]

7. (20 points) Given the power series $f\ (x)$ below,

   (a) Determine the derivative series $f\ '(x)$ and write it in Sigma notation.

   (b) Determine all values of x for which the derivative series $f\ '(x)$ converges.

$$f\ (x) = \sum_{k=1}^{\infty} \frac{2}{k^2 3^k}(x-1)^k$$