# Mathematics 122 Fall 2002

**Exam 3**

**October 29, 2002**

_____
*Name*

**Technology used:** _____ Directions:

Be sure to include in-line citations, including page numbers if appropriate, every time you use a text or notes or technology. Include a careful sketch of any graph obtained by technology in solving a problem. **Only write on one side of each page.**

## The Problems

1. (20 points) Do **one** of the following.

   (a) The base of a solid is the region in the first quadrant bounded by the graphs of $y = x^2$ and $y = \sqrt{x}$. The cross sections of the solid perpendicular to the $x$ - axis are semicircles whose diameters lie in the $xy$ - plane. Find the volume of this solid.

   (b) Find the volume of the solid generated by revolving the region bounded by the curve $y = \frac{4}{x^3}$, the vertical line $x = 1$ and the horizontal line $y = \frac{1}{2}$ about the line $x = 2$.

2. (20 points) Do **one** of the following.

   (a) A rock climber is about to haul up 21 pounds of equipment that has been hanging beneath her on 90 feet of rope that weighs 0.3 $\frac{\text{lb}}{\text{ft}}$. How much work will it take?

   (b) A storage tank is a right circular cylinder 20 feet long and 8 feet in diameter with a **horizontal** axis. If the tank is half full of olive oil weighing 57 $\frac{\text{lb}}{\text{ft}^3}$, find the work done in emptying the tank through an outlet that is 6 feet above the top of the tank.

3. (20 points) Do **one** of the following.

   (a) Use integration by parts to prove the following reduction formula is true.

$$\int \sec^n(x)\, dx = \frac{1}{n-1}\sec^{n-2}(x)\tan(x) + \frac{n-2}{n-1}\int \sec^{n-2}(x)\, dx.$$

   (b) Use the Useful Information for Sequences provided below to find the formula for

$$\sum_{k=0}^{n} k\, 5^k$$

4. (10 points each ) Set up any **four** of the following to the point where the problem can be finished by citing a formula from the Integral Table Handout. Be sure to cite the appropriate formula(s). **Do not use a calculator.**

   (a)

$$\int \frac{1}{1+\sqrt{y}}\, dy$$

(b)

$$\int x^3 \cos\left(x^2\right)\, dx$$

(c)

$$\int \frac{\sin(t)}{(3+\cos(t))^2}\, dt$$

(d)

$$\int x\sqrt{x+2}\, dx$$

(e)

$$\int \frac{x}{9+4x^4}\, dx$$

(f)

$$\int \frac{x+3}{\sqrt{x^2+2x-8}}\ dx$$

**Useful Information about Sequences**

| | | |
|---|---|---|
| $D_k\left[k^{\underline{n}}\right] = nk^{\underline{n-1}}$ | | $D_k\left[c^k\right] = (c-1)\,c^k$ |
| If $a(k) = k^{\underline{n}}$, then $A(k) = \frac{1}{n+1}k^{\underline{n+1}}$ | | If $a(k) = c^k$, then $A(k) = \frac{1}{c-1}c^k$ |
| If $D_k[A(k)] = a(k)$, then $\sum_{k=0}^{n} a(k) = A(k)\Big\|_0^{n+1}$ | | |
| | | |

$$\sum_{k=0}^{n} U(k)\ D_k[V(k)] = U(k)\,V(k)\Big|_0^{n+1} - \sum_{k=0}^{n} V(k+1)\ D_k[U(k)] \quad \text{(Discrete Integration by Parts)}$$