Mathematics 181A **Fall 2008**

Final Exam

December 19, 2008

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Name

Technology used: ______________________________________ **Only write on one side of each page.**

- Show all of your work. Calculators may be used for numerical calculations and answer checking only.

Do any ten (10) of the following.

1. [15 points] Write out one Riemann Sum with $n = 5$ and $\|P\| = 2$ for the function $f(x) = x^2 - x$ on the interval $[0, 6]$ .You do not need to simplify any of the algebra.

2. [10, 5 points] Here is a Riemann sum for a function f on an interval $[a, b]$: $R_n = \frac{1^7+2^7+3^7+\cdots+n^7}{n^8}$

   (a) What are f and $[a, b]$?

   (b) Use the definition of the definite integral to evaluate the limit $\lim_{n\to\infty} \frac{1^7+2^7+3^7+\cdots+n^7}{n^8}$.

3. [15 points] The region in the first quadrant enclosed by the coordinate axes, the curve $y = \ln(x)$, and the line $y = 1$ is revolved around the y-axis to generate a solid. Find the volume of the solid.

4. [15 points] Find the length of the enclosed loop $x = t^2$, $y = (t^3/3) - t$ where t starts at $-\sqrt{3}$ and ends at 0.

5. [15 points] Evaluate the following integral

$$\int \frac{x^4 + 3x^3 + 4x^2 + 13x + 3}{x^3 + 4x} dx$$

6. [15 points] The population of the world was estimated to be 3 billion in 1959 and 6 billion in 1999. What would an exponential model of population growth predict the population to be in 2008? [The actual population was $6,747,510,603$ as of 17:44 GMT (EST+5) Dec 16, 2008 – data from the US Census Bureau's population clock.]

7. Recall that $\int_a^b u\ dv = uv]_a^b - \int_a^b v\ du$.Use this and the fact that $\int_0^\infty t^2 e^{-t}\ dt = 2$ to evaluate the improper integral $\int_0^\infty t^3 e^{-t}\ dt$

   (a) [5 points] Antiderivative

   (b) [10 points] Limit(s)

8. [5, 5, 5 points] Give examples of the following.

   (a) A diverging infinite sequence.

   (b) A converging infinite series.

   (c) An improper integral with at least 3 improprieties.

9. [15 points] Determine the radius of convergence, center, values of x where the series converges absolutely, and values of x where the series converges conditionally for $\sum_{n=0}^{\infty}(-1)^n \frac{(x-2)^n}{4^n\sqrt{n+1}}$

10. [15 points] Find the Taylor series generated by the function $f(x) = \cos(x)$ at $x = a$ where $a = \pi/2$. Be careful. Use the full process for finding a Taylor series (taking derivatives of all orders). **Do not** use the Maclaurin series for $\cos(x)$ since that is not centered at $a = \pi/2$.

11. [15 points] Starting with the Maclaurin series for $\sin(x)$ in the "Useful Information" section below, approximate the value of the integral $\int_0^1 t\sin\left(t^4\right)\,dt$.Use an error bound (not your calculator) to prove your approximation has an error of magnitude less than 10^{-6}.

12. [15 points] Prove that $\lim_{n\to\infty}\frac{x^n}{n!}=0$ for every number x. [Hint: the Maclaurin series for e^x converges for every number x.]

Useful Information

- The Taylor Series for $f(x)$ at $x=a$ is $\sum_{n=0}^{\infty}\frac{1}{n!}f^{(n)}(a)(x-a)^n$
- The error bound for a converging alternating series $\sum_{k=1}^{\infty}(-1)^{k+1}u_k$ is $|L-S_n|<u_{n+1}$
- $\sin(x)=\sum_{n=0}^{\infty}(-1)^n\frac{1}{(2n+1)!}x^{2n+1}$ converges to $\sin(x)$ for every real number x.
- $e^x=\sum_{n=0}^{\infty}\frac{1}{n!}x^n$ for every real number x.