Mathematics 181D — Spring 2008

Exam 4

April 17, 2008

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Name

Technology used: ____ **Only write on one side of each page.**

- Show all of your work. Calculators may be used for numerical calculations and answer checking only.

Examination Problems

1. [5, 5, 5 points] Determine whether the following sequences converge or diverge. Find the limit of each convergent sequence. Explain your answer.

   (a) $a_n = \frac{(-1)^{n+1}}{\sqrt{2n-1}}$.

   (b) $a_n = \left(1 - \frac{1}{n}\right)^{n^2}$

   (c) $a_n = \sqrt[n]{n^5}$

2. [10 points] Do **one** (1) of the following.

   (a) Use partial fractions to find the exact sum of the following series.

   $$\sum_{n=1}^{\infty} \frac{6}{(2n-1)(2n+1)}$$

   (b) Find the values of x for which the following geometric series converges. In addition, find the sum of the series (as a function of x) for those values of x.

   $$\sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^{n-1} (x-3)^{n-1}$$

   (c) Use a geometric series to express the number $0.\overline{7} = 0.77777777\cdots$ as the ratio of two integers. [Hint: this number is (better and better) approximated by 0.7, 0.77, 0.777, etc.]

3. [15 points] Do **one** (1) of the following.

   (a) Use the integral test to determine if the following series converges or diverges. Give reasons and show your work.

   $$\sum_{n=1}^{\infty} \frac{e^{-n}}{1+e^{-2n}}$$

   (b) The P-series $\sum_{n=1}^{\infty} \frac{1}{n^3}$ can be shown to converge using the integral test. Bound the error in using $S_4 = 1 + \frac{1}{2^3} + \frac{1}{3^3} + \frac{1}{4^3} = \frac{2035}{1728}$ to approximate the actual limit of this infinite series.

4. [15 points each] Do **four** (4) of the following but make sure **at least one** (1) of them is an alternating series.

   Which of the following series converge absolutely, which converge conditionally, and which diverge? Give reasons and show your work.

(a) $\sum_{n=1}^{\infty} \frac{1}{1+\ln(n)}$

(b) $\sum_{n=1}^{\infty} \frac{(n!)^2}{(2n)!}$

(c) $\sum_{n=1}^{\infty} (-1)^{n+1} \left(\sqrt[n]{10} \right)$

(d) $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{\sqrt{n}}{n^2+n}$

(e) $\sum_{n=1}^{\infty} \left(1 - \frac{3}{n}\right)^n$

(f) $\sum_{n=1}^{\infty} \frac{(-3)^n}{n!}$

(g) $\sum_{n=1}^{\infty} \frac{n}{(\ln(n))^n}$