Mathematics 181B — Spring 2012

Exam 3

March 29, 2012

_____
Name

Technology used: _____ **Only write on one side of each page.**

- Show all of your work. Calculators may be used for numerical calculations and answer checking only.

"It is hard to know what you are talking about in mathematics, yet no one questions the validity of what you say. There is no other realm of discourse half so queer." – J. R. Newman

Problems

1. [15 points] Write the following integral (which has multiple improprieties) as the sum of improper integrals each of which has exactly one impropriety which occurs at a limit of integration. **Do not evaluate any of the integrals.**
$$\int_{-\infty}^{\infty} \frac{1}{x\left(x-4\right)} dx$$

2. [15 points] Evaluate the following integral. Show all of your work. You **may** use the table of integrals.
$$\int \frac{x^4+81}{x\left(x^2+9\right)^2}\, dx$$

3. [15 points each] Do any (2) of the following.

   (a) Use Simpson's rule to estimate the minimum number of subintervals needed to approximate the following integral with an error of magnitude less than 10^{-4}.The Error bound formula is: $\left|\int_a^b f\left(x\right) dx - S_n\right| \leq \frac{M(b-a)^5}{180n^4}$.
   $$\int_0^3 \frac{1}{\sqrt{x+1}}\, dx$$

   (b) Evaluate
   $$\int_2^{\infty} \frac{2}{t^2-1}\, dt$$

   (c) Evaluate
   $$\int_0^5 \frac{1}{\left(x-2\right)^{4/3}}\, dx$$

4. [10 points each] For **two** (2) of the following, determine if the sequence converges or diverges. If it converges, determine its limit.

   (a) $b_n = \left(1-\frac{2}{n}\right)^n$

   (b) $a_n = \frac{\ln(n)}{\sqrt{n}}$

   (c) $c_n = \ln\left(n\right) - \ln\left(2n+1\right)$

5. [10 points each] Do **two** (2) of the following.

   (a) Does $\sum_{n=1}^{\infty} \ln\left(\frac{n}{2n+1}\right)$ converge or diverge? If it converges, find the sum.

   (b) Does $\sum_{n=3}^{\infty} \frac{5^{n+1}}{10^n}$ converge or diverge? If it converges, find the sum.

   (c) For what values of x does the following infinite series converge?

$$\sum_{n=0}^{\infty} \left(-\frac{1}{2}\right)^n (x-3)^n$$

6. **Extra Credit**. [5 points] The formula for the sequence whose terms are $0, 2, 0, 2, 0, 2, 0, 2, 0, 2, \cdots$ is $a_n = 1+(-1)^n$ .What is the formula for the sequence whose terms are $-3, 23, -3, 23, -3, 23, -3, 23, \cdots$?