Mathematics 181–D **Spring 2007**

Exam 5/Final

May 11, 2007

____
Name

Technology used:____________________________________ Directions:

- Be sure to include in-line citations every time you use technology.
- **Only write on one side of each page.**
- **When given a choice, specify which problem(s) you wish graded.**

Exam 5

Do any six (6) of the following problems (15 points each)

1. Replace the polar equation below by an equivalent Cartesian equation and the Cartesian equation below by an equivalent polar equation.

   (a) $\cos^2\theta = \sin^2\theta$

   (b) $(x-5)^2 + (y+2)^2 = 29$

2. Identify the symmetries of the curve $r^2 = 9\sin(3\theta)$ and make a careful sketch using a table (A sketch with no supporting table will get zero credit).

3. Consider the graph of $r = 1 + 2\sin(\theta)$.

   (a) Find all values of θ between 0 and 2π for which the graph passes through the origin.

   (b) Compute the slope of the curve at **one** of these points $[r, \theta]$.

   (c) Sketch the curve along with the tangent line whose slope you have found.

4. Find the area of the region inside the lemniscate $r^2 = 6\cos(2\theta)$ and outside the circle $r = \sqrt{3}$.

5. Find the length of the curve given by the polar coordinate equation $r = 8\sin^3(\theta/3)$, $0 \le \theta \le \pi/4$.

6. The area of the region inside the cardioid curve $r = 1 + \cos\theta$ and outside the circle $r = \cos\theta$ is **not** $\frac{1}{2}\int_0^{2\pi}\left[(1+\cos\theta)^2 - \cos^2\theta\right]\,d\theta = \pi$.

   (a) Explain why this is not the area.

   (b) Write a definite integral that specifies the actual area but **do not evaluate** that integral.

7. Can anything be said about the relative lengths of the curves $r = f(\theta)$, $\alpha \le \theta \le \beta$ and $r = 2f(\theta)$, $\alpha \le \theta \le \beta$? Explain.

8. Use Riemann Sums to give a detailed derivation of the formula, $A = \int_\alpha^\beta \frac{1}{2}[f(\theta)]^2\,d\theta$, for the area bounded by the polar curve $r = f(\theta)$, $\alpha \le \theta \le \beta$.

Cumulative Final Examination

Section One (15 points each): Do any two (2) of the following

I.1. Evaluate the following limit by interpreting it as the limit of Riemann sums for a particular definite integral and then evaluating that integral.

$$\lim_{n\to\infty}\left(e^{1/n}+e^{2/n}+\cdots+e^{n/n}\right)\frac{1}{n}$$

I.2. Evaluate

$$\int_0^{\pi/2}\frac{3\sin(x)\cos(x)}{\sqrt{1+3\sin^2(x)}}\,dx$$

I.3.

$$\text{Find}\quad\frac{dy}{dx}\quad\text{if}\quad y=\int_{\arctan(x)}^{\pi/4}e^{\sqrt{t}}\,dt$$

Section Two (15 points each): Do any two(2) of the following

II.1. Do **one** (1) of these volume problems.

(a) A round hole of radius $\sqrt{3}$ft is bored through the center of a solid sphere of radius 2 ft. Find the volume of material removed from the sphere.

(b) A solid is generated by revolving about the x-axis the region bounded by the graph of the positive continuous function $y=f(x)$, the x-axis, and the fixed line $x=a$, and the variable line $x=b$, $b>a$. The volume of this solid for every such b is b^2-ab. Find $f(x)$.

II.2. Find the area of the surface generated by revolving the curve $x=\sqrt{4y-y^2}$, $1\le y\le 2$ about the y-axis.

II.3. **Set up, but do not evaluate**, the integrals for M, M_y, and M_x needed to find the center of mass of a thin, flat plate covering the region enclosed by the parabola $y=x^2$ and the line $y=2x$ if the density function is $\delta(x)=1+x$. Use vertical strips.

Section Three (15 points each): Do any two (2)of the following

III.1. Evaluate $\int\frac{v+3}{v^3-4v}\,dv$.

III.2. Evaluate $\int\frac{\sin(5t)\,dt}{1+[\cos(5t)]^2}$.

III.3. Does this improper integral converge or diverge? Why?

$$\int_0^{\infty}\frac{1}{x^2(1+e^x)}dx$$

Section Four (10 points): Do any one (1) of the following

IV.1. Use a comparison test to prove: If $\sum_{n=1}^{\infty}a_n$ and $\sum_{n=1}^{\infty}b_n$ are divergent series of nonnegative numbers, then $\sum_{n=1}^{\infty}a_nb_n$ must also diverge.

IV.2. Use a comparison test to prove: If $\sum_{n=1}^{\infty}a_n$ and $\sum_{n=1}^{\infty}b_n$ are convergent series of nonnegative numbers, then $\sum_{n=1}^{\infty}a_nb_n$ must also converge.