

March 27, 2007

Name

Technology used: _____ Directions:

- Be sure to include in-line citations every time you use technology. Include a careful sketch of any graph obtained by technology in solving a problem. **Only write on one side of each page. When given a choice, specify which problem(s) you wish graded.**

The Problems

1. (10 points) Express the integrand of the following integral as a sum of partial fractions with undetermined coefficients. **Do not solve for the coefficients or evaluate the integrals.**

$$\int \frac{x^9 - 6x^5 + 7}{x(x+3)^4(x^2+4)^2(x^2+x+1)^2} dx$$

2. [15 points each] Do two (2) of the following three (3) problems about integrals.

(a) Evaluate the integral

$$\int \frac{v^2 dv}{(1-v^2)^{5/2}}$$

(b) Find the volume of the solid obtained by revolving the region bounded by $y = \frac{3}{\sqrt{3x-x^2}}$, $0.5 \leq x \leq 2.5$ about the x -axis.

(c) Make a substitution first and then evaluate the integral

$$\int \frac{e^{4t} + 2e^{2t} - e^t}{e^{2t} + 1} dt$$

3. [15 points] Estimate the minimum number of subintervals needed to approximate $\int_0^1 \sin(x+1) dx$ with an error of magnitude less than 10^{-5} using Simpson's Rule. The error bound formula is $|E_S| \leq \frac{M(b-a)^5}{180n^4}$.

4. [15 points] Do one (1) of the following two (2) problems.

(a) Determine if the following integral represents a number. If it does, find it. If it does not, explain why.

$$\int_{-2}^3 \frac{1}{(x+1)^2} dx$$

(b) Write the following integral (which has multiple improprieties) as the sum of improper integrals each of which has exactly one impropriety which occurs at a limit of integration. Evaluate any **one** of these integrals.

$$\int_{-2}^{\infty} \frac{1}{x(x-4)} dx$$

5. [8, 7 points] Explain whether the following infinite sequences converge or diverge and determine, with explanation, the limit of any that converge.

(a) $a_n = 3 + 2(-1)^n$

(b) $b_n = \frac{4n^4 + 3n}{2n^4 + 1000n^3}$

6. [15 points] Write out the first 5 terms of the sequence of partial sums of the infinite series $\sum_{k=1}^{\infty} (-1)^k \frac{1}{n(n+1)}$.