Mathematics 181 – **C** **Spring 2007**

Exam 1

February 6, 2007

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Name

Technology used: _____

- – **Only write on one side of each page.**
  - – Show all of your work. Calculators may be used for numerical calculations and answer checking only.
  - – Be sure to include in-line citations every time you use technology and Include a careful sketch of any graph obtained by technology in solving a problem.

Do any six (6) of the following problems

1. (10 points) Do **one** (1) of the following:

   (a) Use the definition of limit (which means you will have ε and δ in your answer) to prove that $\lim_{x\to 3}(5x-8)=7$.

   (b) Use the graphical interpretation of limits (using vertical and horizontal windows) to show that if $f(x)=\begin{cases} x+1, & \text{if } x\leq 1 \\ x-1, & \text{if } x>1 \end{cases}$ then $\lim_{x\to 1} f(x)$ does not exist. Give details.

2. (8, 7 points) Without using a calculator, evaluate **both** of the following indefinite integrals

   (a) $\int \left(5\cos(x)-\frac{2}{\sqrt{1-x^2}}+\sec(x)\tan(x)+\frac{1}{x^{2/3}}\right)\,dx$

   (b) $\int \frac{6+2\sqrt{t}}{t}\,dt$

3. (10 points) Let P be a partition of the interval $[0,3]$ into n subintervals of equal length. Use sigma notation to write the lower sum L for the function $f(x)=4-x^3$ using this partition. **Do not simplify.** [Remember that the lower sum is the Riemann sum that is less than or equal to every other Riemann sum.]

4. (15 points) Without using a calculator, do **one** (1) of the following:

   (a) Find the derivative $H'(x)$ of
   $$H(x)=\int_1^{e^{3x}} \frac{\ln(t)}{t^5+3}\,dt.$$

   (b) Find a function f that satisfies the equation: $\tan(x)=\int_2^x \sqrt{7+f(t)}\,dt$.

5. (10 points each) Use substitution to evaluate **both** of the following

   (a)
   $$\int \frac{\sin(\sqrt{x})\cos^7(\sqrt{x})}{\sqrt{x}}\,dx$$

(b)

$$\int \frac{x\left(\arctan\left(x^2\right)\right)^5}{1+x^4}\,dx$$

6. (15 points each) Do **both** of the following.

   (a) The base of a solid sits on the region in the xy-plane bounded by the x-axis and the graph of the semicircle $y = \sqrt{9-x^2}$. If cross sections perpendicular to the x-axis are rectangles with height twice as great as their base, what is the volume of the solid?

   (b) Find the volume of the solid obtained when the region in the first quadrant bounded by the curve $x = y - y^3$, the line $x = 1$ and the line $y = 1$ is rotated about the line $y = 1$.

   (c)

   (d) $\int_0^1 2\pi\left(-y+1\right)\left(y-y^3\right)dy = \frac{7}{30}\pi$

   (e) $\int 2\pi\left(-y+1\right)\left(y-y^3\right)dy = 2\pi\left(\frac{1}{5}y^5 - \frac{1}{4}y^4 - \frac{1}{3}y^3 + \frac{1}{2}y^2\right)$

   (f) $\frac{1}{60}y^2\left(y\left(3y\left(4y-5\right)-20\right)+30\right) = \frac{1}{60}y^2\left(y\left(3y\left(4y-5\right)-20\right)+30\right) = \frac{1}{60}y^2\left(12y^3-15y^2-20y+30\right)$