Mathematics 181 – B **Fall 2007**

Exam 1

September 18, 2007

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Name

Technology used:______________________________ Directions:

- Be sure to include in-line citations every time you use technology.
- Include a careful sketch of any graph obtained by technology in solving a problem.
- **Only write on one side of each page.**

Do any six (6) of the following problems

1. [8, 7 points] Do any **two** (2) of the following three options.

   (a) What are the antiderivatives of the following functions?

      i. $\sin(kx)$
      ii. $\sec^2(kx)$
      iii. $\frac{1}{\sqrt{1-x^2}}$
      iv. $\frac{1}{1+x^2}$
      v. e^{kx}

   (b) The accompanying table gives data for the velocity of a vintage sports car accelerating from 0 to 142 miles per hour in 36 seconds (10 thousandths of an hour). Use rectangles to estimate how far the car traveled during the 36 seconds it took to reach 142 mi/h. If you use your calculator on this problem be sure to also write out the formula for the estimate.

Time (h)	Velocity (mi/h)	Time (h)	Velocity (mi/h)
0.0	0	0.006	116
0.001	40	0.007	125
0.002	62	0.008	132
0.003	82	0.009	137
0.004	96	0.001	142
0.005	108		

   (c) Are any of the following equal? If so, which. You do not need to actually add up any of them.

      i. $\sum_{k=3}^{100}(k-1)^2$
      ii. $\sum_{k=17}^{118}(k-19)^2$
      iii. $\sum_{k=-96}^{1}(k-1)^2$
      iv. $\sum_{k=-1}^{97}(k+3)^2$

2. [15 points] Use the Riemann Sum process to:

   (a) Find a formula for the upper sum for the function $f(x) = 1 + x^3$ over the interval $[0, 2]$ obtained by partitioning $[0, 2]$ into n equal subintervals. [Useful fact: $\sum_{k=1}^{n} k^3 = \frac{1}{4}n^2(n+1)^2$]

   (b) Take the limit of these sums as $n \to \infty$ to calculate the area under the curve over $[0, 2]$.

3. [15 points] Use the fact that $f(x) = 1 + x^3$ is monotone increasing over the interval $[0, 2]$ to find an error bound for the estimate in part a. of the previous problem. Include any pertinent figures and write your answer as a function of n (the number of subintervals).

4. [15 points] Example 1 of Section 5.3 in the textbook explains why the function below is not integrable on the interval $[0, 1]$.

   (a) Explain why it is true that for any partition P of the interval $[0, 1]$ it is possible to select points c_k in two different ways: one where the Riemann sum adds up to 1 and another where the Riemann sum adds up to 0.

   (b) Explain why this is enough to show that the function is **not integrable.**

$$f(x) = \begin{cases} 1, & \text{if } x \text{ is rational} \\ 0, & \text{if } x \text{ is irrational} \end{cases}$$

5. [15 points] Do both of the following.

   (a) Express as a definite integral where P is a partition of $[-3, -1]$.

$$\lim_{\|P\| \to 0} \sum_{k=1}^{n} \left(\tan^2(7c_k)\, e^{-c_k} \right) \Delta x_k$$

   (b) Does this limit exist? Why? If you think it exists, do not bother to compute it.

6. [15 points] The Domination property of Table 5.3 in Section 5.3 of the text applies to integrable functions and reads

$$f(x) \geq g(x) \text{ on } [a, b] \text{ implies } \int_a^b f(x)\, dx \geq \int_a^b g(x)\, dx$$

   (a) Use the full Riemann Sum process to explain why this is true.

7. [15 points] Compute the following function values by using a well-known area formula.

   (a) $F(2)$ where $F(x) = \int_0^x \sqrt{4 - x^2}\, dx$

   (b) $F(-2)$ where $F(x) = \int_0^x \sqrt{4 - x^2}\, dx$

   (c) Find $\frac{dy}{dx}$ for the function

$$y = \int_{\tan(x)}^{0} \frac{dt}{1 + t^2}$$