Mathematics 121-A — Spring 2000

Exam 4

March 10, 2000

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Name

Technology used: ____

Directions: Include a careful sketch of any graph obtained by technology in solving a problem. **Only write on one side of each page.**

The Problems

1. (8 points each) Evaluate the following derivatives. Do **not** simplify.

   (a) $y = (x^3+1)\sin(x)$

   (b) $y = \frac{x^2+1}{1+\sec(x)}$,

   (c) $T = (2s^{-4} + 3s^{-2} + 2)^{-6}$,

   (d) $f(x) = \sqrt{5x-8}$,

   (e) $g(x) = \ln(\sin(x^2+7))$,

   (f) Evaluate

   $$\frac{d^4}{dx^4}[4x^3 - 2x^5]$$

2. (10 points) Use the quotient rule and the derivatives of $\sin(x)$ and $\cos(x)$ to show

   $$\frac{d}{dx}\cot(x) = -\csc^2(x)\,.$$

3. (8 points each) Use the following table of outputs for the functions f, f', g and g' to compute the indicated derivatives.

x	f	f'	g	g'
1	-2	-0.5	3	4
2	-4	-1	1	-3
3	0	0	9	2
4	3	2	4	0

   (a) Find $F'(4)$ if $F(x) = f(x) - 3g(x)$

   (b) Find $H'(2)$ if $H(x) = 2 + f(g(x))$.

4. (15 points) Do **one** of the following

(a) Suppose a pebble is thrown vertically upward from the top of a 800 foot high building with an initial velocity of 32 feet per second.

i. Find the height of the pebble at $t = 3$ s.

ii. Find the velocity of the pebble at $t = 3$ s.

iii. Find the velocity of the pebble when it hits the ground.

iv. Find the maximum height of the pebble.

(b) An object moves along a coordinate line with position at time t (seconds) given by $x(t) = t + 2\cos(t)$ (meters). Find those times t from 0 to π when the object is moving forward and also slowing down.

5. (11 points) Do **one** of the following.

(a) Each of the following limits represents the derivative of some function f at some number c. State f and c in each case.

i.

$$\lim_{h\to 0} \frac{\sqrt{1+h}-1}{h}$$

ii.

$$\lim_{h\to 0} \frac{(2+h)^3-8}{h}$$

iii.

$$\lim_{x\to 1} \frac{x^9-1}{x-1}$$

(b) Prove for a differentiable function f and a constant c,

$$\frac{d}{dx}\left[cf(x)\right] = c\frac{d}{dx}\left[f(x)\right]$$

by using the (limit) definition of derivative and the fact

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$