Mathematics 121-A — Spring 2000

Final Exam

May 8, 2000

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Name

Technology used: ______________________________

Textbook/Notes used: ______________________________

Directions: Be sure to include in-line citations, including page numbers if appropriate, every time you use a text or notes or technology. Include a careful sketch of any graph obtained by technology in solving a problem. **Only write on one side of each page.**

The Problems

1. (4 points each) Find the derivatives of the following functions. Do not simplify. Write your answers in the space provided.

   (a)

   $$P(x) = 7x^9 - 3x^5 + 12x^2 - 4x^{1/2} + 31$$

   (b)

   $$f(x) = \sin(x^2 + 5x + 7)\sec(3x^5 - 12x^2 - 4)$$

   (c)

   $$g(x) = \frac{5x^4 - 16x^2 + 4}{\tan(9x^3 + 1)}$$

   (d)

   $$C(x) = \left(\sqrt{2}\right)^{\pi} \quad \text{(Think about this one.)}$$

   (e)

   $$y = x^3 + (x^2 + [x + \sin(x)]^2)^3$$

2. (10 points) Use the definition of derivative to find $f'(x)$ if $f(x) = \frac{1}{2x+4}$.

3. (10 points) Use the definition of limit to **prove** that

   $$\lim_{x \longrightarrow 4} (3x - 15) = -3.$$

4. (10 points) Determine the values of a and b that make the following function continuous at $x = 0$. Show that all three criteria of continuity are satisfied.

$$f(x) = \begin{cases} e^x - a, & x = 0 \\ b, & x > 0 \\ \frac{x^3 - 4x}{x^2 - 2x}, & x < 0 \end{cases} .$$

5. (5 points each) Find the following indefinite integrals.

   (a) $\int \frac{\cos(\theta)}{3} \, d\theta$

   (b) $\int x \left(x + \sqrt{x}\right) dx$

   (c) $\int \left(2x^2 + 5\right)^2 dx$

   (d) $\int \frac{x^2 + 3x - 1}{x^4} \, dx$

6. (15 points) Do **one** of the following.

   (a) A ship with a long anchor chain is anchored in 22 meters of water. The anchor chain is being wound in at the rate of 20 meters per minute, causing the ship to move toward the spot directly above the anchor resting on the seabed. The hawshole—the point of contact between ship and chain—is located 2 meters above the waterline. At what speed is the ship moving when there are exactly 26 meters of chain still out?

   (b) At what rate is the surface area of a cube changing at the instant when the volume of the cube is decreasing at a rate of 9 cubic centimeters per minute and the length of a side is decreasing at the rate of 4 centimeters per minute?

   (c) A highway patrol plane flies 3 miles above a level straight road at a steady ground speed of 120 miles per hour. The pilot sees an oncoming car and determines with radar that the line-of-sight distance from the plane to the car is 5 miles and decreasing at the rate of 160 miles per hour. Find the car's speed along the highway.

7. (15 points each) Do **two** of the following.

   (a) An **open-topped** rectangular box with square base has volume 500 cm^3. Find the dimensions of the box that minimize the total area A of its base and four sides.

   (b) An apartment complex has 200 units. When the monthly rent for each unit is \$600, all units are occupied. Experience indicates that for each \$20-per-month increase in rent, 5 units will become vacant. Each rented apartment costs the owners of the complex \$80 per month to maintain. What monthly rent should be charged to maximize the owner's profit? What is the maximum profit?

   (c) Of all the pairs of numbers that sum to 11.5, which pair has the largest product?

8. (15 points) Find the **exact** area bounded by the graph of $y = \frac{4}{(1+x^2)}$ and the interval $[-1, 1]$ on the x-axis.

9. (20 points) Given the function

$$f(x) = \frac{-x}{x-1},$$

List all of the following information in the spaces provided. You need not sketch the function.

   (a) Critical Points ____________________

(b) Second order critical points ________________

(c) Intervals where f is increasing ________________

(d) Intervals where f is decreasing ________________

(e) Intervals where f is concave up ________________

(f) Intervals where f is concave down ________________

(g) Horizontal asymptotes ________________

(h) Vertical asymptotes ________________

(i) Oblique asymptotes ________________