Mathematics 121-A **Exam 5** **Spring 2006**

May 8, 2006

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Name

Directions:

- **Only write on one side of each page.**
- **Use terminology correctly.**
- **Show your work: answers that can be obtained from a calculator will not receive credit.**
- **Partial credit is awarded for correct approaches so justify your steps.**

Do any five (5) of the following.

1. (20 points) Evaluate any four (4) of the the following indefinite integrals.

   (a)
$$\int \frac{1}{7x}\, dx$$

   (b)
$$\int \left(2e^x + 3\sec(x)\tan(x)\right)\, dx$$

   (c)
$$\int \left(\sec^2(x) + \frac{1}{x}\right)\, dx$$

   (d)
$$\int \frac{1}{t^2}\left(\frac{5}{t^3} + \frac{2}{t}\right)\, dt$$

   (e)
$$\int \frac{1}{\sqrt{9-9x^2}}\, dx$$

   (f)
$$\int 4\left(\cos(x)\right)^3\left(-\sin(x)\right)\ \ dx$$

2. (20 points) Do **one** (1) of the following.

   (a) Use the First Fundamental Theorem of Calculus to find the exact area of the region bounded by the graph of $y = x^3 + 3x^2$,the x- axis, and the vertical lines $x = 1$ and $x = 4$.

   (b) Starting from rest, an airplane has a constant acceleration of k ft per second squared while moving down the runway. What is the acceleration of the plane if it requires 900 feet of runway before lifting off at $88\frac{\text{ft}}{\text{s}}$? $[a(t) = v'(t) = s''(t)]$

3. (20 points) On May 7, 1992, the space shuttle *Endeavor* was launched on mission STS-49. The table below, provided by NASA, gives the velocity data for the shuttle between liftoff and the jettisoning of the solid rocket boosters. Use the data in the table to form a Riemann sum that estimates the height above the earth's surface of the space shuttle *Endeavor*, 62 seconds after liftoff. **Simplify your sum.**

Event	Time (s)	Velocity (ft/s)
Launch	0	0
Begin roll maneuver	10	185
End roll maneuver	15	319
Throttle to 89%	20	447
Throttle to 67%	32	742
Throttle to 104%	59	1325
Maximum dynamic pressure	62	1445
Solid rocket booster separation	125	4151

4. ($20 = 6 + 7 + 7$ points) The following limit gives the exact area of a region in the plane as the limit of Riemann sums of a function f. The Riemann sums were obtained by partitioning an interval $[a, b]$ into n subintervals of equal size Δx and using the right endpoints of each subinterval. **DO NOT EVALUATE** the limit.

$$\lim_{n\to+\infty} \sum_{k=1}^{n} \left[\left(3 + \frac{4}{n}k\right)^5 - 2\left(3 + \frac{4}{n}k\right)^2 + 5 \right] \frac{4}{n}.$$

   (a) What is Δx ?
   (b) What is the function f ?
   (c) What is the interval $[a, b]$?

5. (20 points) Use the Second Fundamental Theorem of Calculus to answer both of the following.

   (a) Find the derivative $G'(x)$, of

$$G(x) = e^x \int_3^x \frac{\tan(t)}{t^2+1}\, dt.$$

   (b) Find the derivative $H'(x)$ of

$$H(x) = \int_1^{e^{3x}} \ln\left(t^2+1\right)\, dt.$$

6. (20 points) The slope $F'(x)$ at various points on a graph is given in the figure below. Each block of the grid is 2 units wide and 2 units high. Use this information to sketch the graph of the antiderivative F of F' that passes through the point $(0, -1)$. **Draw your sketch on this graph.**

   i. What is the the value of $F'(6)$?
   ii. Use your graph to estimate the value of $F(6)$?s

(a)

Figure 1: