Mathematics 121-A Exam 4 Spring 2006

April 20, 2006

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Name

Directions:

- **Only write on one side of each page.**
- **Use terminology correctly.**
- **Show your work: answers that can be obtained from a calculator will not receive credit.**
- **Partial credit is awarded for correct approaches so justify your steps.**

Required Problem

A.1. (10 points) Verify that the function $f(x) = \frac{1}{x+1}$ satisfies the hypotheses of the Mean Value Theorem on the interval $[0, 2]$ and find all numbers c between 0 and 2 for which $\frac{f(b)-f(a)}{b-a} = f'(c)$.

Do any four (4) of the following.

B.1. (15 points) Let n be a positive integer.

(a) Show that there is a number c between 0 and x for which

$$\frac{(1+x)^n - 1}{x} = n(1+c)^{n-1}$$

(b) Use the result above to compute

$$\lim_{x\to 0} \frac{(1+x)^n - 1}{x}$$

B.2. (15 points) Draw a curve on the coordinate system below that represents the derivative of the function whose graph is shown.

B.3. (15 points) Determine the intervals of increase and decrease and concavity of the following function. Find the critical numbers and second-order critical numbers, and asymptotes. **Do Not Sketch the graph.**

$$f(x) = 1 + 2x + \frac{18}{x}$$

B.4. (15 points) Find all vertical and horizontal asymptotes of the function f given below. Find where f is increasing and decreasing, determine intervals of concavity, and locate all critical points and points of inflection. Draw a **careful** sketch of the graph of f including such special features as cusps and vertical tangents if they exist.

$$\begin{aligned} f(x) &= \frac{3x+5}{7-x} \\ f'(x) &= \frac{26}{(-7+x)^2} \\ f''(x) &= -\frac{52}{(-7+x)^3} \end{aligned}$$

B.5. (15 points) Let $P(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$ be a polynomial with $a_n \neq 0$. Let $L = \lim_{x\to-\infty} P(x)$ and $M = \lim_{x\to\infty} P(x)$. Fill in the missing entries in the following table.

Sign of a_n	n	L	M
$+$	even		$+\infty$
$+$	odd	$-\infty$	
$-$	even		
$-$	odd		$-\infty$

Do any two (2) of the following

C.1. (15 points) Compute **one** (1) of the following

(a)

$$\lim_{x\to\infty}\left(1+\frac{1}{2x}\right)^{3x}$$

(b)

$$\lim_{x\to 0}\left(\frac{1}{\sin(2x)}-\frac{1}{2x}\right)$$

C.2. (15 points) A truck is 250 miles due east of a sports car and is traveling west at a constant speed of 60 mi/hr. Meanwhile, the sports car is going north at 80 mi/hr. When will the truck and the car be closest to each other and what is the minimum distance between them? [Hint: minimize the square of the distance.]

C.3. (15 points) According to postal regulations, the girth plus the length of a parcel sent by fourth-class mail may not exceed 108 inches. What is the largest possible volume of a rectangular parcel with two square sides that can be sent by fourth-class mail. See the figure on the blackboard.

C.4. (15 points) [This problem is from section 4.7] A commuter train carries 600 passengers each day from a suburb to a city. It now costs $5 per person to ride the train. A study shows that 50 additional people will ride the train for each $0.25 reduction in fare. What fare should be charged to maximize total revenue?