Mathematics 121-A Exam 2 Spring 2006

February 23, 2006

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Name

Directions:

- **Only write on one side of each page.**
- **Use terminology correctly.**
- **Show your work: answers that can be obtained from a calculator will not receive credit.**
- **Partial credit is awarded for correct approaches so justify your steps.**

Do any seven (7) of the following.

Do not use a calculator to justify any problem except number 2.

1. [10 points] Use an ε, δ proof to show that $\lim_{x \to 4} (-2x+1) = -7$.

2. [10 points] Given the limits $\lim_{x \to 1^+} \frac{2}{x-1}$, $\lim_{x \to 1^+} \frac{x^2-2x+1}{x-1}$, and $\lim_{x \to 6} \frac{\tan(\pi/x)}{x-1}$

   (a) In your own words, explain why the first limit does not exist but the other two do.

   (b) Evaluate the last two limits.

3. [15 points] Use the definition of continuity to determine if the function $f(x) = \begin{cases} \frac{x^2-9}{x+3}, & \text{if } x < 3 \\ 6, & \text{if } x = 3 \\ 5x-9, & \text{if } x > 3 \end{cases}$ is continuous at $x = 3$.

4. [15 points] Do all of the following:

   (a) Simplify $\log_2(16)\log_3\left(\frac{1}{27}\right)$

   (b) Find the numbers x that solve the equation $\frac{e^{x^2}}{e^{x+6}} = 1$

   (c) If $\log_{\sqrt{b}}(106) = 2$ what is $\sqrt{b-25}$?

5. [15 points] Compute the derivative of $f(x) = \frac{x}{x+3}$ by evaluating the limit $\lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$.

6. [20 points] Use the derivative rules for the following.

   (a) Find $f'(x)$ if $f(x) = 3x^4 - 7x^2 + \frac{2}{x} + \sqrt{x}$.

   (b) If $h(x) = (x^3 + x^2 + 1)(3x^2 - 4)$ use the product rule to find $h'(x)$.

   (c) Find $\frac{dy}{dx}$ if $y = \frac{x^3+x}{2x^2-1}$

   (d) Find $\frac{d^3y}{dt^3}$ where $y = 2t^4 - 3t^3 + 4t - 6$

7. [15 points] Do **one** (1) of the following

   (a) Does the function $f(x) = x^3 + 2x^2 - 3x$ satisfy the equation $y''' + y'' + y' = 3x^2 + 10x + 7$?

   (b) Find an equation for a tangent line to the graph of $f(x) = \frac{3x+5}{1+x}$ that is perpendicular to the line $2x - y = 1$. [There are two.]