

Problems on differentials

1. The volume V of a right circular cylinder is related to the radius r and height h of the cylinder by $V = \pi r^2 h$.

(a) Find the linear relation among the differentials dV , dr , and dh .

$$\text{Answer: } dV = \pi r^2 dh + 2\pi r h dr$$

(b) Use your result from (a) to deduce a relation among percent changes in V , r , and h .

$$\text{Answer: } \frac{dV}{V} = \frac{dh}{h} + 2\frac{dr}{r}$$

(c) If the height and radius of a cylinder are each increased by 1%, by what percent does the volume increase?

$$\text{Answer: } 3\%$$

(d) If the height of a cylinder is increased by 1%, how must the radius be changed to keep volume constant?

$$\text{Answer: } \text{Decrease radius by } 1/2\%$$

2. Consider a consumer who can purchase different amounts of three commodities: apples, bananas, and cherries. Let a , b , and c be the amount purchased of each (measured in pounds). A simple model used by economists assigns a utility U (in units we'll call *utils*) to each bundle (a, b, c) the consumer can purchase according to the formula

$$U = k a^{1/2} b^{1/6} c^{1/3}$$

where $k = 1$ util/lb (to keep units consistent).

- (a) Find the linear relation among differentials dU , da , db , and dc .
- (b) Use your result from (a) to deduce a relation among percent changes in U , a , b , and c .
- (c) For which of the commodities would 1% increase in amount purchased lead to the smallest change in utility? What is the percentage change in utility corresponding to a 1% increase in the amount purchased of that commodity?

3. The volume V of a sphere is related to the radius r of the sphere by $V = \frac{4}{3}\pi r^3$.

(a) Find the linear relation between the differentials dV and dr .

(b) Suppose volume and radius are changing in time t . Use your result from (a) to get a relation between the rate of change in V with respect to t and the rate of change in r with respect to t .

(c) Suppose air is being pumped into a balloon at the rate 0.2 cubic meters per second. How fast is the radius changing at the time when the radius is 1.5 meters?

4. Consider the relation $z = \cos(xy)$.
- Find the linear relation among the differentials dx , dy , and dz .
 - Consider a level curve in the xy -plane for z constant so $dz = 0$. Use your relation from (a) to get a formula for the slope dy/dx of a level curve.
 - Use your result in (b) to compute the slope of the level curve that passes through the point $(x, y) = (5, 2)$.
5. Suppose x , y , and z are related by a function $z = f(x, y)$.
- Find the linear relation among the differentials dx , dy , and dz . Note that this will involve partial derivatives of f .
 - Consider a level curve of f with z constant so $dz = 0$. Use your relation from (a) to get a general formula for the slope dy/dx of a level curve in terms of partial derivatives of f .
 - What condition must hold in order for you to use your result from (b) to compute dy/dx ?
6. Problem 48 in Section 12.6
7. Problem 49 in Section 12.6