Mathematics 433 **Fall 2010** **Problems to Turn in: 4**

October 21

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Name

Directions: Be sure to include in-line citations, including page numbers if appropriate, every time you use the results of discussion, a text, notes, or technology. **Only write on one side of each page.**
"Mathematicians do not study objects, but relations among objects; they are indifferent to the replacement of objects by others as long as relations do not change. Matter is not important, only form interests them."
— Henri Poincaré

Problems

1. **You Must Do This Problem** Let $\tau = (a_1, a_2, \cdots, a_k)$ be a cycle of length k.

   (a) Prove that if σ is any permutation, then $\sigma\tau\sigma^{-1} = (\sigma(a_1), \sigma(a_2), \cdots \sigma(a_k))$.

   (b) Let μ be a cycle of length k. Prove there is a permutation σ such that $\sigma\tau\sigma^{-1} = \mu$.

2. Do all three of the following. Prove that any element in S_n (where $n \geq 3$) can be written as a finite product of

   (a) the transpositions $(12), (13), \cdots (1n)$.

   (b) the transpositions $(12), (23), \cdots, (n-1, n)$

   (c) the two distinct cycles $(12), (12 \cdots n)$

3. Let $\sigma \in S_X$. For any $x, y \in X$, we say $x \sim y$ if there is an integer n such that $\sigma^n(x) = y$.

   (a) Prove that $\sim$ is an equivalence relation.

   (b) Let $x \in X$ and $\sigma \in S_X$ and define the **orbit of** x **under** σ to be the set $\mathcal{O}_{x,\sigma} = \{\sigma^n(x) : n \in \mathbb{Z}^+\}$. Prove that $\mathcal{O}_{x,\sigma}$ is the equivalence class of x under the equivalence relation $\sim$.
   [Note that this gives us a way to use a group to partition a set. We will see much more about this later.]

   (c) Let $X = 1, 2, 3, 4, 5, 6$ be the set of faces of a cube where, as viewed from a fixed location, $1, 2, 3, 4, 5, 6$ denote the front, right, back, left, top, and bottom faces respectively. Compute the orbit of face 1 under the element $\alpha = (13)(24) \in S_4$

   (d) Using the same set X as above, find all of the equivalence classes $\mathcal{O}_{x,\sigma}$ in X where $\sigma = (124) \in S_4$.