Mathematics 433 **Fall 2010** **Problems to Turn in:** 02

Due September 24

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Name

Directions: Be sure to include in-line citations, including page numbers if appropriate, every time you use the results of discussion, a text, notes, or technology.

Only write on one side of each page.

"*Simple solutions seldom are. It takes a very unusual mind to undertake analysis of the obvious.*" — Alfred North Whitehead

Problems

1. You must do this problem. Do **two** of the following.

   (a) Prove the Leibiz rule for $f^{(n)}(x)$ where $f^{(n)}(x)$ is the nth derivative of f; that is, show that for every positive integer n

   $$(fg)^{(n)}(x) = \sum_{k=0}^{n} \binom{n}{k} f^{(k)}(x)\, g^{(n-k)}(x)$$

   (b) Let X be a set and $\mathcal{P}(X) = \{A : A \subset X\}$ the **power set** of X (the set of all subsets of X). Prove that for every positive integer n, any set with exactly n elements has a power set with 2^n elements.

   (c) Use induction to compute the determinant of $A = \begin{bmatrix} 2 & -1 & & & & \\ -1 & 2 & -1 & & & \\ & -1 & 2 & -1 & & \\ & & & \ddots & & \\ & & & -1 & 2 & -1 \\ & & & & -1 & 2 \end{bmatrix}$. (All other entries in this tridiagonal matrix are zero.)

2. Prove that the First Principle of Mathematical Induction implies the Second Principle of Mathematical Induction. [You may use anything we already know about induction.]

3. Let $P(A)$ be the **power set** of the set A.That is, $P(A)$ is the set of all subsets of A. Show that for any set (including infinite sets) A it is not the case that A is in one-to-one correspondence with $P(A)$. [Hint: the infinite case is much more interesting than the finite case and the method of proof indicated in this hint will work for both. Let $\phi : A \to P(A)$ be any one-to-one function and show it cannot be onto by considering the subset of A consisting of all elements a that are not in their image, $\phi(a)$.]

4. The textbook's statement of the First Principle of Mathematical Induction uses $n_0 > 0$ as a base case. Prove that the specical case of this that we proved in class (i.e., the case where $n_0 = 1$) implies the more general case given in the book.