Proof VS-2

Accepted **Not Accepted**

I affirm this work abides by the university's Academic Honesty Policy.

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Print Name, then Sign

- First due date **Thursday, November** 6.
- Turn in your work on a separate sheet of paper with this page stapled in front.
- Do not include scratch work in your submission.
- There is to be **no collaboration** on any aspect of developing and presenting your proof. Your only resources are: you, the course textbook, me, and pertinent discussions that occur **during class.**
- Follow the Writing Guidelines of the Grading Rubric.
  (http://math.ups.edu/~bryans/Current/Fall_2008/290inf_Fall2008.html#tth_sEc5.1)
- Retry: Only use material from the relevant section or earlier.
- Retry: Start over using a new sheet of paper.
- Retry: Restaple with new attempts first and this page on top.

"Experience is what enables you to recognize a mistake when you make it again." (Earl Wilson)

VS-2 (Section PD)
Suppose $S = \{\vec{v}_1, \vec{v}_2, \vec{v}_3, \cdots, \vec{v}_{p-1}, \vec{v}_p, \vec{v}_{p+1} \cdots \vec{v}_m\}$ is an orthonormal basis for $\mathbf{C}^m$ and let $V = \langle\{\vec{v}_1, \vec{v}_2, \vec{v}_3, \cdots, \vec{v}_{p-1}, \vec{v}_p\}\rangle$ be the subspace of $\mathbf{C}^m$ spanned by the first p vectors in S and $W = \langle\{\vec{v}_{p+1} \cdots \vec{v}_m\}\rangle$ be the subspace of $\mathbf{C}^m$ spanned by the last $m - p$ vectors in S.

1. Quote the theorem from our textbook that tells us that $\mathbf{C}^m = V \oplus W$.
2. Prove that if $\vec{w} \in W$, then $\vec{w}$ is orthogonal to every vector in V.
3. Prove that if $\vec{x}$ is orthogonal to every vector in V, then $\vec{x} \in W$.

Notes

- Because W satisfies the two properties (2 and 3) above, it is called the **orthogonal complement** of V in $\mathbf{C}^m$ and is usually written $V^\perp$.
- Professor Beezer has proved that
  1. Every subspace, V, of $\mathbf{C}^m$ has a basis
  2. That basis can be extended to a basis of $\mathbf{C}^m$ and
  3. The Gram-Schmidt procedure can transform any basis into an orthonormal basis.

  Your work along with these details proves the theorem

  Theorem 1 *If V is a suspace of $\mathbf{C}^m$ then $\mathbf{C}^m = V \oplus V^\perp$*
