Smith **Math 181** **Fall 2008**

Project 1

I affirm this work abides by the university's

- Due **Thursday, October** 9 at the beginning of class.
- Turn in the final version of your work on a separate sheet of paper with this page stapled in front.
- Do not include scratch work in your submission.
- Follow the Writing Guidelines of the Grading Rubric on the last page of the course information sheet. (http://math.ups.edu/~bryans/Current/Fall_2008/181inf_Fall2008.html)
- You may use any technology that you like (e.g., calculators, Mathematica, MATLAB, etc.).
- You may work with others in solving these problems but there is to be **no collaboration on the written exposition** of the solutions.
- Include a reference paragraph at the beginning of your paper either affirming the work is completely yours or citing each resource you use: names of participants in discussions (other than in-class discussions), technological tools, reference texts employed, and anything else other than your own thoughts.

"'Know thyself?' If I knew myself, I'd run away." – Johann von Goethe

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Project 1: Two cylinders of radius 3 inches intersect at right angles. What is the volume of the intersection?

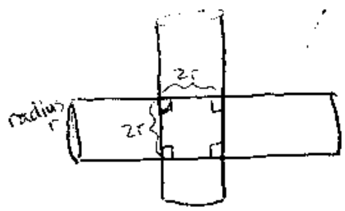
This is essentially the shape of the "U joint" in an older car's drive train. It is the location where the energy from the spinning drive shaft (which is oriented front to back in the car) is converted into energy to spin the axles (which are oriented perpendicular to the drive shaft).

Most people find it helps a lot to see a physical model of this shape. I have one in my office you may see, or you can form one yourself using chalk or cork.

Project 1: Two cylinders of radius 3 inches intersect at right angles. what is the volume of the intersection?

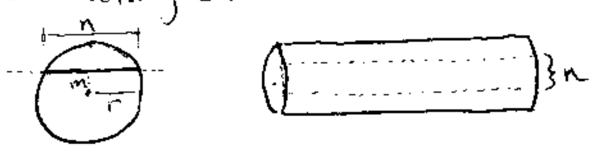
In solving this problem, I sought the help of Professor Bryan Smith who helped me visualize the intersection and helped me develop the technique for solving for the volume. I also used the textbook University Calculus, by Hass Weir and Thomas, as a reference in order to find the definition of the volume of a solid of known integrable cross sectional area

If we look at the intersection of two cylinders of the same radius that intersect perpendicularly to each other from a viewpoint straight above the intersection, we can see that the widest cross section of the intersection is a square.

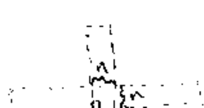


The length of each side of this cross-section will be equivalent to the diameter of each cylinder which is 2 times radius r.

If we decide to take a plane and cut one of the cylinders parallel to its center axis a distance m above the center axis, the cross section made by the plane and the cylinder will be a rectangle with a width n and a length as long as the cylinder

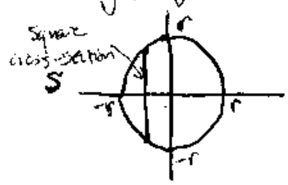


If we repeat this planar cut a distance m above the center axis of the other cylinder with the same radius r, we would see that this cross-section will also have a width n. If these two slices, one from each cylinder, are intersected perpendicularly to each other, we can see that the intersection they create is also a square. This square will have sides that equal n.

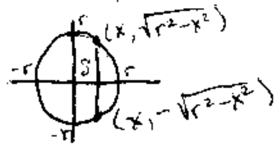


This means that the solid made by the intersection of two perpendicular cylinders of radius r are going to have square cross sections at any distance m above or below ~~parallel to~~ the center of the solid when planes are cut parallel to the center axes of the cylinders. (Prof. Smith, 10/8/08

If we cut a cylinder with a radius with a plane perpendicular to its center axis, we see that the cross-section made by this cut is a circle with a radius r. If we look at our solid of intersection from the side, we can also see that our square slices are laying perpendi to a circle cross-section. We can graph this circular cross-section just as the base of a solid w the square cross-sections intersecting perpendicular to the x-axis. We will put the center of circle at the origin. (Prof. Smith, 10/8/08)



The line in the circle represents one side of a typical square cross-section of the solid. Because the endpoints of the square side lie on the circle cross-section, we can find the length of the square side using the equation for a circle: $x^2+y^2=r^2$. We need to rewrite the equation of the circle as a function y in terms of x in order to find the coordinates of the square length's endpoints. We do this by subtracting both sides by x^2, which leaves $y^2=r^2-x^2$. When can then take the square root of both sides to find y by itself: $y=\pm\sqrt{r^2-x^2}$. It is plus and minus the square root of r^2-x^2 in order to cover the top and bottom of the circle. We can now label the endpoints of the square cross-section with coordinates in terms of x.



By subtracting the y-coordinate of the endpoint in the ~~first~~ fourth quadrant from the y-coordinate of the endpoint in the first quadrant, we find the length of side S equals $2\sqrt{r^2-x^2}$.

$$S = \sqrt{r^2-x^2} - (-\sqrt{r^2-x^2}) = 2\sqrt{r^2-x^2}$$

We can use the formula for finding the area of a square (Area = (side length)^2) in order to find the area of the square cross sections. When we square the side length S(x) we get the area A(x

$$A(x) = [S(x)]^2 = (2\sqrt{r^2-x^2})^2 = 4(r^2-x^2)$$

According to the textbook University Calculus (by Hass, Weir, Thomas), the volume of a solid of known integrable cross-sectional area $A(x)$ from $x=a$ to $x=b$ is the integral of A from a to b.

$$V=\int_a^b A(x)\,dx$$

Our a value will be $-r$ and our b value will be r because the circle lies on the planes from $x=-r$ to $x=r$. So we can now integrate our cross-sectional area $A(x)$ from $-r$ to r in order to find the volume of the solid. We set up the integral: $V=\int_{-r}^{r} A(x)\,dx$

$$=\int_{-r}^{r}(4)(r^2-x^2)\,dx$$

For our particular problem the radius is three, so we replace r with 3.

$$V=\int_{-3}^{3} 4(3^2-x^2)\,dx$$

$\int_{-3}^{3}(3^2-x^2)dx$

We move the constant four to the front of the integral and separate ~~3^2-x^2~~ into two integrals

$$V=4\left[\int_{-3}^{3}3^2dx-\int_{-3}^{3}x^2dx\right]$$

$$=4\left[\int_{-3}^{3}9\,dx-\int_{-3}^{3}x^2dx\right]$$

We then integrate the $9dx$ and the x^2dx ~~from~~ on their intervals $[-3,3]$ ~~using the fund. Thm of Calc part 2~~ and solve for the volume using the Fund. Thm of Calculus part 2 (University Calculus)

$$V=4\left[(9x]_{-3}^{3}-\left(\tfrac{x^3}{3}\right]_{-3}^{3}\right]$$

$$=4\left[9(x)_{-3}^{3}-\tfrac{1}{3}(x^3]_{-3}^{3}\right]$$

$$=4(9(3-(-3))-\tfrac{1}{3}(3^3-(-3)^3)$$

$$=4[9(6)]-4\left[\tfrac{1}{3}(27-(-27))\right]$$

$$=4(54)-4\left[\tfrac{1}{3}(54)\right]$$

$$=216-\tfrac{216}{3}$$

$$\begin{array}{r}54\\ \times\ 4\\ \hline 216\end{array}$$

We find the common denominator and subtract. To simplify the fraction, we divide 432 by 3.

$$=\tfrac{648}{3}-\tfrac{216}{3}$$

$$=\tfrac{432}{3}$$

$$\begin{array}{r}216\\ \times\ 3\\ \hline 648\end{array}\qquad \begin{array}{r}648\\ -216\\ \hline 432\end{array}$$

$$\begin{array}{r}144\\ 3\overline{)432}\\ 3\\ \hline 13\\ 12\\ \hline 12\end{array}$$

After integrating our definite integral, and calculating the results, we obtain a value of 144. We add the units of inches^3 to give the exact volume of 144 in^3. The volume of the intersection between two cylinders of radius 3 inches intersecting at a right angle is 144 in^3.