6, 2.5, 1

Smith **Math 181** **Fall 2008**

Project 1

I affirm this work abides by the university's Acade[illegible]

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Print Name, then Sign

- Due **Thursday, October** 9 at the beginning of class.
- Turn in the final version of your work on a separate sheet of paper with this page stapled in front.
- Do not include scratch work in your submission.
- Follow the Writing Guidelines of the Grading Rubric on the last page of the course information sheet.
  (http://math.ups.edu/~bryans/Current/Fall_2008/181inf_Fall2008.html)
- You may use any technology that you like (e.g., calculators, Mathematica, MATLAB, etc.).
- You may work with others in solving these problems but there is to be **no collaboration on the written exposition** of the solutions.
- Include a reference paragraph at the beginning of your paper either affirming the work is completely yours or citing each resource you use: names of participants in discussions (other than in-class discussions), technological tools, reference texts employed, and anything else other than your own thoughts.

"'Know thyself?' If I knew myself, I'd run away." – Johann von Goethe

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Project 1: Two cylinders of radius 3 inches intersect at right angles. What is the volume of the intersection?

This is essentially the shape of the "U joint" in an older car's drive train. It is the location where the energy from the spinning drive shaft (which is oriented front to back in the car) is converted into energy to spin the axles (which are oriented perpendicular to the drive shaft).

Most people find it helps a lot to see a physical model of this shape. I have one in my office you may see, or you can form one yourself using chalk or cork.

Finding the Volume of the Intersection of Two Cylind .

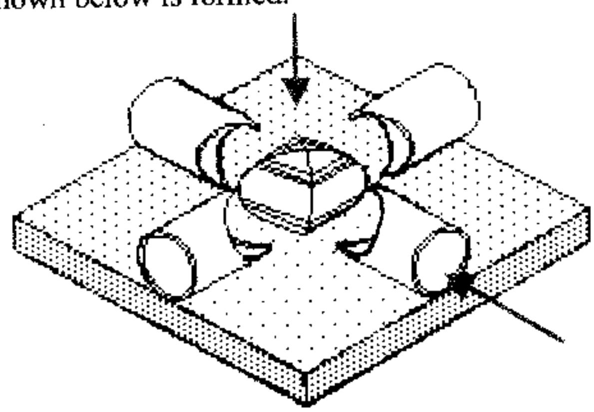
References:

The ideas and concepts presented in this report were discussed with the following people, Professor Bryan Smith, Becka Lowen, and Vu Nugyen, along with my self. The concepts and the formulas used were found in the Hass Weir Thomas, *University Calculus,* textbook. I have used the ideas and formulas discussed to solve this problem. The image and graph below were provided by, Foster Manufacturing Company (http://www.flash.net/~fmco) and by Web Math (http://www.webmath.com/gcircle.html).

Problem:

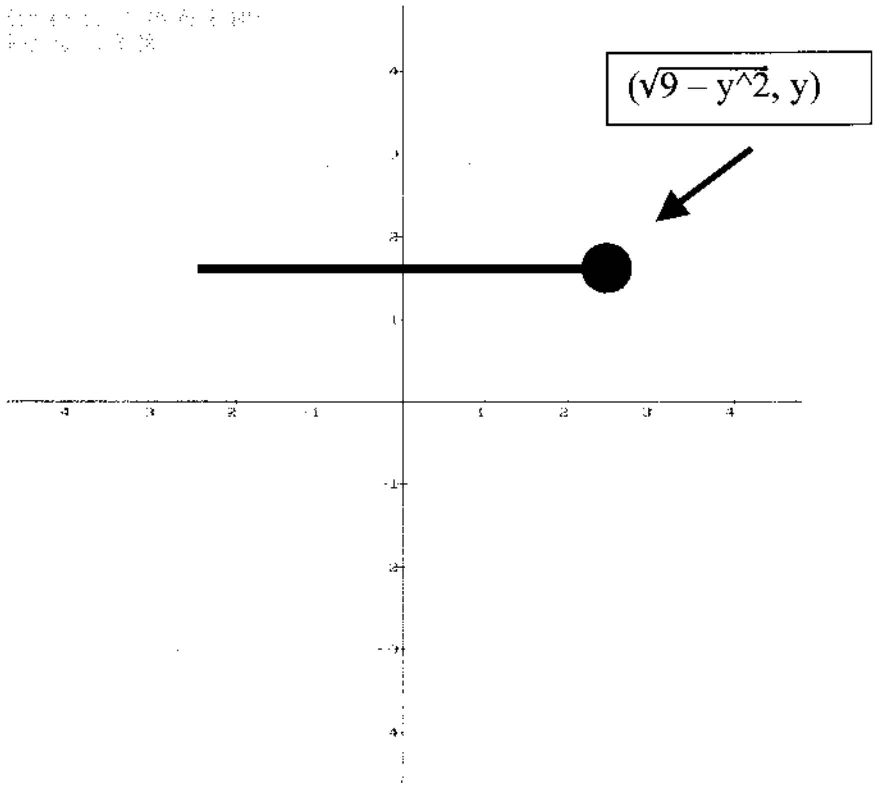
Two cylinders of radius 3 inches intersect at right angles. What is the area of the intersection?

To solve this problem we must first try to visualize exactly what this "intersection" shape looks like. When we intersect two cylinders at right angles, the volumetric shape shown below is formed.



If we look at this shape from the side (through one of the cylinders, "red arrow"), we see a circle. This circle has radius r = 3 inches. If we look down on the object (blue arrow) we see a square. This square will be very useful later. In order to find its volume, first we place the intersection in an x-y plane. To do this, we graph a circle using the equation

$x^2 + y^2 = r^2$, with r = 3 inches. This is the circle that we saw when looking at the object from the side. Thus, the final equation for our circle will be $x^2 + y^2 = 9$. The graph of this circle is shown below.



To find the volume of the intersection we can use integration and the cross sectional areas of slices cut out of the object. For this to work, we want our solid to have cross sectional areas, which are planar or flat. This way we can find the area of the cross-section with relative ease. The volumes by slicing method states that if the cross-section of the solid S at each point x, between the interval [a, b] is a region R(x) of area A(x), and A is a continuous function of x, we can define and calculate the volume of the solid S as a definite integral (Hass, Weir, Thomas). Remember that integrals can be approximated, by using Riemann sums. So, if we let the norm of the partition ||P|| go to zero and as the number of subintervals n reaches infinity, we get the lim n→infinity of ∑ from k=1 to n, of A(x) Δx sub k. This Riemann sum is equal to the ∫ from a to b of A(x)dx. In this definite integral, dx represents the height of the square (seen from above the object). We will use this integral formula to calculate the volume of the volumetric shape.

In order to find the volume, we need to have a function to integrate. We find this function f(y) by solving the circle equation for the variable x.

Li: why is this "2" here?

$$x^2 + y^2 = r^2, \quad r = 3$$

$$\Rightarrow \quad x = 2\sqrt{9 - y^2}$$

This function serves as length and width of our square cross-section. Since the area of a square with side length x, as stated earlier, is equal to x^2, we get that the square, cross-sectional area of the volumetric shape is equal to $2\sqrt{9 - y^2} = f(y)$.

Finally, we integrate the function f(y) by applying the formula for finding volumes by slicing.

$$V = \int A(y)\,dy \qquad \text{(with limits of integration -3 to 3)}$$

$$\Rightarrow \quad V = \int \left(2\sqrt{9 - y^2}\right)^2 dy \qquad \text{(with limits of integration -3 to 3)}$$

$$\Rightarrow \quad V = 4 \int (9 - y^2)\,dy \qquad \text{(limits of integration -3 to 3)}$$

$$\Rightarrow \quad V = 4\,(9y - (y^3)/3) \qquad \text{(evaluated from -3 to 3)}$$

$$\Rightarrow \quad V = 4[(\,9(3) - 3^3/3\,) - (\,9(-3) - (-3)^3/3)]$$

$$V = 4\,(36)$$

$$V = 144 \text{ in}^3$$

In conclusion, by integrating the equation for the cross-sectional area of the intersection of two cylinders, with a radius of 3 inches, we can find the total volume of the shape. In this case we found that the volume is equal to 144 cubic inches.