Mathematics 433 **Fall 2007** **Problems to Turn in: 5**

October 26, 2007

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Name

Directions: Be sure to include in-line citations, including page numbers if appropriate, every time you use the results of discussion, a text, notes, or technology. **Only write on one side of each page.**

These and all previous problems MUST be turned in before October 13

"*To those who do not know mathematics it is difficult to get across a real feeling as to the beauty, the deepest beauty of nature. If you want to learn about nature, to appreciate nature, it is necessary to understand the language that she speaks in*". -Richard Feynman (1918-1988)

Problems

1. Do the following.
   (a) Prove that the relation x is conjugate to y in a group G is an equivalence relation on G.
   (b) Describe the elements a whose conjugacy class (that is, whose equivalence class) consists of the element a alone.
2. Classify groups of order 6 by analyzing the following three cases.
   (a) G contains an element of order 6.
   (b) G contains an element of order 3 but none of order 6.
   (c) All elements of G have order 1 or 2.
3. Let $\phi : G \to G'$be a homomorphism and let H' be a subgroup of G'.Denote the inverse image $\phi^{-1}(H') = \{x \in G : \phi(x) \in H'\}$ by $\tilde{H}$. Prove
   (a) $\tilde{H}$ is a subgroup of G.
   (b) If H' is a normal subgroup of G' then $\tilde{H}$ is normal in G.
   (c) $\tilde{H}$ contains the kernel of ϕ
   (d) The restriction of ϕ to $\tilde{H}$ defines a homomorphism whose kernel is $\ker(\phi)$.
4. Prove that every group whose order is a power of a prime p contains an element of order p.