Mathematics **Fall 2006** **Project 1**

Due September 1, 2006

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Name

Directions: Whenever appropriate, use in-line citations, including page numbers and people consulted when you present information obtained from discussion, a text, notes, or technology. **Only write on one side of each page.**

"No, no, you're not thinking, you're just being logical." -Niels Bohr, physicist (1885-1962)

Project Description

Discrete domain functions (sequences) have derivative formulas and rules that are analogous to the formlas and rules of interval domain functions. We will explore a few of them in this handout.

For example, consider the sequence given by the function $a(n) = n^2$ which is more precisely defined by

$$\begin{aligned} a &: \mathbf{N} \cup \{0\} \rightarrow \mathbf{R} \\ a &: n \rightarrow a(n) = n^2 \end{aligned}$$

Then, $D_n[a(n)] = \frac{a(n+1)-a(n)}{1}$ becomes

$$\begin{aligned} D_n[a(n)] &= \frac{a(n+1)-a(n)}{1} \\ &= a(n+1)-a(n) \\ &= (n+1)^2 - n^2 \\ &= [(n+1)+n][(n+1)-n] \\ &= (2n+1)(1) \\ &= 2n+1 \end{aligned}$$

Thus we can say that the line segment joining terms n and $n+1$ of this sequence has slope $2n+1$.As a specific example, the line segment joining terms 3 and 4, $a(3) = 9$, $a(4) = 16$ has slope $m = 2(3)+1$ because $\frac{16-9}{1} = 2(3)+1$.

So we now have a **formula** for discrete derivatives: $D_n[n^2] = 2n+1$ and in a similar fashion we can compute $D_n[n^3] = 3n^2+3n+1$ and $D_n[n^4] = 4n^3+6n^2+4n+1$.

Here are a couple of 'nicer' examples:

1. First, let p be a positive integer and define $n^{\underline{p}} = n(n-1)(n-2)(n-3)\cdots(n-p+1)$. As an example, $n^{\underline{3}} = n(n-1)(n-2)$ and $(n+1)^{\underline{3}} = (n+1)(n)(n-1)$. If we now compute the discrete derivative of the sequence $a(n) = n^{\underline{3}}$ we get the "familiar" formula $D_n[n^{\underline{3}}] = 3n^{\underline{2}}$

$$\begin{aligned} D_n[a(n)] &= \frac{a(n+1)-a(n)}{1} \\ &= a(n+1)-a(n) \\ &= (n+1)^{\underline{3}} - n^{\underline{3}} \\ &= (n+1)(n)(n-1) - n(n-1)(n-2) \\ &= n(n-1)((n+1)-(n-2)) \\ &= 3n(n-1) \\ &= 3n^{\underline{2}} \end{aligned}$$

2. Let $a\left(n\right)=3^{n}$. Then, $D_{n}\left[3^{n}\right]=2\cdot3^{n}$

$$\begin{aligned} D_{n}\left[a\left(n\right)\right] &= \frac{a\left(n+1\right)-a\left(n\right)}{1} \\ &= a\left(n+1\right)-a\left(n\right) \\ &= 3^{n+1}-3^{n} \\ &= 3^{n}\left(3-1\right) \\ &= 2\cdot3^{n} \end{aligned}$$

Homework Problems

1. Show

$$\begin{aligned} D_{n}\left[n^{\underline{2}}\right] &= 2n^{\underline{1}} \\ &= 2n \end{aligned}$$

2. Show

$$D_{n}\left[n^{\underline{4}}\right]=4n^{\underline{3}}$$

3. Show

$$D_{n}\left[n^{\underline{5}}\right]=5n^{\underline{4}}$$

4. Show

$$D_{n}\left[n^{\underline{p}}\right]=pn^{\underline{p-1}}$$

5. Show

$$D_{n}\left[2^{n}\right]=2^{n}$$

6. Show

$$D_{n}\left[4^{n}\right]=3\cdot4^{n}$$

7. Show

$$D_{n}\left[5^{n}\right]=4\cdot5^{n}$$

8. Show

$$D_{n}\left[r^{n}\right]=\left(r-1\right)r^{n}$$