MATH 433, Abstract Algebra
Fall 2006

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Contents

1 Introduction

The only formal prerequisite for this course is Linear Algebra (Math 290/232). This means you should be familiar not only with the standard methods and techniques for thinking about and solving proof-oriented mathematical problems but also with the basics of developing and writing proofs.

Although there are few prerequisites, Abstract Algebra 433 is a senior level course. Faculty in mathematics consider this course a capstone for those of you pursuing either a career in secondary education or graduate studies in mathematics.

By taking Math 433, you will acquire a deeper knowledge of linear algebra as well as learn the fundamentals of group theory. This latter topic provides an exceptional example of the true power of algebra: take a useful, concrete example and abstract its basic concepts to such a level that the abstraction can be applied to many new situations. In particular, group theory has applications to quantum theory, molecular structure, symmetry (ranging from the structure of crystals to the mathematics describing attempts at "Grand Unified Theories" of everything), coding theory, the structure of complex numbers, the topology of the universe, and much more.

Although most of the group theory we will cover is identical with that in chapters 0-11 of our text, we will not be closely following the structure of that book. In particular, we will be reviewing, and extending linear algebra as we weave it into our studies of group theory. Thus, you should not think of our text as the course bible, but rather as your primary resource for filling in details of the material covered in class. I also recommend that you take the time to find and use additional references. In particular, there are an abundance of useful books in the library and mathematics reading room.

For an official description of this course, see MATH 433 Syllabus[4].

By the time we finish this course, you should also have refined the following skills.

- read a mathematical text for content and deep understanding (see "How to Study" [7] for an excellent description of how to read mathematics and other efficient ways to study),
- analyze a given problem to determine which tools should be used in its solution,
- use a variety of strategies to determine and prove a solution of the given problem, and
- follow accepted mathematical style to present an accurate and carefully written formal proof of your solution.

2 Course Information

2.1 Textbook

The textbook is *Contemporary Abstract Algebra*, 5th Ed, Joseph A. Gallian, ©2002, Houghton Mifflin Company.

Since most of you are majoring in either mathematics or science, you should consider buying *Scientific Notebook* or some other technical word processor. Almost all such products have student editions that are significantly less expensive than the standard editions.

You should already feel comfortable with the basic approaches to doing a mathematical proof. However, if you would like to have a resource at hand, you might consider buying one of the many books on "how to do proofs". I recommend "The Nuts and Bolts of Proofs" but one of the other books

listed below might appeal to your learning style better. The links point to Amazon.com but you might find better prices elsewhere on the web.

- "The Nuts and Bolts of Proofs", Antonella Cupillari [10]
- "How to Read and do Proofs", Solow [12]
- "Thinking Mathematically", Mason/Burton/Stacey [11]
- "Mathematical Thinking: Problem-Solving and Proofs", West and D'Angelo [14]
- "How to Prove It: A Structured Approach", Velleman [13]
- "Proofs and Fundamentals: A First Course in Abstract Mathematics", Bloch [9]

2.2 Basic Information

You can find information pertinent to all of my classes at the link below and, once there, information specific to this class by clicking on the Math 433 link.

`http://math.ups.edu/~bryans/` [1]

2.2.1 Logistics

Because of the construction on Thompson Hall, the rooms for my office and our class might change during the semester. Currently those rooms are as listed below.

Professor Bryan Smith	(Temp) Thompson 321E	879-3562	bryans[at]ups.edu
Office Hours		Tuesday	2:00 - 2:50 P.M.
		Wednesday	3:00 - 4:30 P.M.
		Friday	1:00 - 1:50 P.M.
Classroom / time	Thompson 139	M,W,F	9:00 - 9:50 A.M.
		Tuesday	8:30 A.M. - 9:20 P.M.

I am also available to meet at other times. If you have trouble meeting during office hours please make an appointment for a better time.

2.3 Day to Day Structure

The class weeks will be typically be structured as follows.

Monday, Wednesday, Friday These days will be devoted to lectures. Very little time, if any, will be given for questions.

Monday On test weeks, Monday during class will be a review session. If you request, I am willing to consider, on a trial basis, having an evening office hour.

Tuesday All examinations are scheduled for Tuesday.

On weeks when there is no examination, Tuesday will be devoted to questions and discussions about the course material (and possibly new material).

2.4 Examinations

Since the homework problems will be used to facilitate deep understanding, tests will be used more as a basic check of your knowledge. Hence, there will be two, "straightforward", semester examinations.

Examination One	Tuesday	September 26
Examination Two	Tuesday	November 7

2.5 Final Examination: Wednesday December 13 at 8:00 A.M.

The final will also be "straightforward" and comprehensive. The final cannot be rescheduled so do not plan plane flights (or anything else) that will conflict with it.

2.6 Writing Projects

Many homework problems will be assigned throughout the semester. They represent a selection of the available problems that highlight important concepts, techniques, or computational skills. Most of these problems will not be collected so you are expected to work as many of them as you feel necessary to master the material.

However, 25 problems will be collected and marked. Of these, 20 of these will be marked for mathematical accuracy. When you present these, you are to assume an audience of your Math 433 peers and provide justifications for every step in your argument that is not *clear* to this audience.

In addition, over the course of the semester, you are to designate 5 of the 25 problems as "writing" problems which I will mark for both mathematical accuracy and clarity of exposition (see below and my web page for some basic guidelines for writing mathematics). Do not turn in more than one of these writing problems in any week since their purpose is to provide feedback as you develop your mathematical writing style. I expect at least 3 of these problems to be turned in by October 19. Credit for any problem is earned when you turn in a complete and accurate solution. If you turn in an incomplete or inaccurate solution, I will make a few comments and return it. You may then re-submit the problem. This process may be repeated. To provide some reinforcement for being timely, you may submit no more than 5 problems per week. On the other hand, if I return a problem but ask for a "verbal" explanation of some point(s), then that verbal discussion does not count as a problem submission.

Feel free to use (or not) any technology that you like (e.g., Scientific Notebook, CABRI, Geometers Sketchpad, calculators, *Mathematica*, MATLAB, etc.). You may also work with others in solving these problems but there is to be no collaboration (other than consulting with me) in the writing of the solutions. Moreover, you **must** cite each resource you use. This includes: technological tools, texts read, participants in discussions and anything else other than your own thoughts. Citations are to occur in the text proper (in-line). Do not use footnotes or endnotes except in exceptional circumstances. In addition, you **m**ust include a reference paragraph at the beginning of your paper either affirming the work is completely yours or citing each resource you use: names of participants in discussions (other than the in-class discussions), technological tools, reference texts employed, and anything else other than your own thoughts. Failure to include references is intellectual theft! Please see the "Academic Honesty" section of the *Logger* to see how serious this issue is to the university community.

These "writing" problems are actually the mathematical equivalent of assigned papers in humanities courses. In particular, be sure to invest adequate time in analysis and research before you completely explain that analysis of the problem. You are also expected to follow standard mathematical style in your presentation. More detailed explanations of that style can be found at this link to my "Journal of Undergraduate Mathematics at Puget Sound" (http://www.math.ups.edu/ bryans/JournalGuidelines.html). You can also peruse any copy of *Mathematics Magazine* or the American Mathematical Association

Monthly to see how authors employ citations and references. You can find copies of these journals in the Mathematics Reading Room, my office or the library. Although you may not have much experience with mathematical writing, you will find it to be quite similar to other expository styles: build an argument by making a sequence of claims and provide logical justifications for each claim. Your papers will be graded both for mathematical content and for written presentation using the grading rubric on the last page of this document.

Since most of you are either science or mathematics majors, you should attempt to use a word processor to write your papers. One possibility is *Scientific Notebook* since it is not only an easy to use technical word processor but also contains a symbolic algebra package that can be useful for analyzing problems. Another, cheaper, approach would be to learn the basics of LaTeX (or TeX) and install the software package MikTeX, see `http://miktex.org` [5], and a front end program such as TeXnicCenter, see `http://miktex.org/Links.aspx`, [6]. This last approach gives you far more control over your mathematical writing but has a steeper learning curve. On the other hand, knowledge of LaTeX is almost a requirement for graduate school in mathematics and science.

2.7 Reading

Developing an ability to read and understand a (relatively) technical piece of writing is a primary goal of this course. This skill is fundamental not only for those who wish a career in science but also for anyone who wishes to be a well-rounded member of society. Hence, careful reading of the texts is an integral part of this course — especially since lectures will not be word-for-word reiterations of the material in the textbook. I recommend multiple readings of the material as we cover it since technical material is difficult to grasp quickly. (See "How to Study"[7] on the course webpage for more details.)

2.8 Course Information Updates

If you wish, I will post (and update) a grade report on your current standing in the class on my university web page. You should keep track of your grades on the various assignments and check them against these reports. If there are any discrepancies they should be dealt with immediately.

To have your information posted you need to print your name, the class (MATH 433), and a code on a sheet of paper. Then sign the paper and physically hand it to me. The code is to be a sequence of up to 23 symbols I can type on a keyboard.

2.9 Total Points

Homework: Normal	46%
Homework: Writing	18%
Examinations	24%
Final Examination	12%

2.10 First Graded Homework Assignment

(Due Friday September 1 at 5:00 P.M.)

1. Look over both my university web page `http://math.ups.edu/~bryans/` [1] and the course webpage for MATH 433 you'll find there.

2. Send an e-mail message to me at bryans [at] ups.edu that contains the information below. Make sure the course number, 433, and your name are in the "Subject" line.

(a) Tell me your mathematical plans after graduation.

(b) Write a paragraph or two detailing your personal learning style. Include any classroom techniques you have found that enhance or block your learning.

References

[1] Bryan Smith's Homepage
`http://math.ups.edu/~bryans/`

[2] Math 433A Course Webpage
`http://math.ups.edu/~bryans/Current/Fall_2006/433Index_Fall2006.html`

[3] Department Calculator Policy
`http://www.math.ups.edu/info/calcpolicy.pdf`

[4] Department Syllabus for MATH 433
`http://www.math.ups.edu/~matthews/Syllabi/MA433Syllabus.pdf`

[5] Location of MikTeX package for using LaTeX `miktex.org`

[6] Development tools for composing LaTeX documents `miktex.org/Links.aspx`

[7] William Rapaport's "How to Study"
`http://www.cse.buffalo.edu/~rapaport/howtostudy.html`

[8] TI-86 Manual
`http://education.ti.com/us/product/tech/86/guide/86guideus.html`

[9] "Proofs and Fundamentals: A First Course in Abstract Mathematics", Bloch
`http://www.amazon.com/exec/obidos/ASIN/0817641114/`

[10] "The Nuts and Bolts of Proofs", Antonella Cupillari
`http://www.amazon.com/exec/obidos/tg/detail/-/0120885093/`

[11] "Thinking Mathematically", Mason/Burton/Stacey
`http://www.amazon.com/exec/obidos/ASIN/0201102382/`

[12] "How to Read and Do Proofs", Solow
`http://www.amazon.com/exec/obidos/ASIN/0471406473/`

[13] "How to Prove It: A Structured Approach", Velleman
`http://www.amazon.com/exec/obidos/ASIN/0521446635/`

[14] "Mathematical Thinking: Problem-Solving and Proofs", West and D'Angelo
`http://www.amazon.com/exec/obidos/ASIN/0130144126/`

3 Math 433 Writing Projects Grading Rubric

Points	Logic and Mathematics
5	Arguments are correct, complete and without inappropriate material.
4	Arguments have one minor error, omission or inappropriate inclusion.
2	Arguments have two minor errors, omissions or inappropriate inclusions.
0	Arguments are more seriously flawed.

Points	Use of Terminology and Notation
3	All technical terms, concepts and notation are used correctly.
2	Arguments have one lapse in terminology and notation
1	There are minor problems with terminology or concepts.
0	There are major problems with terminology or concepts.

Points	Written Presentation
2	Follows citation requirements and all other writing guidelines.
1	Follows almost all of the guidelines with only one or two minor lapses.
0	Has more lapses in following the guidelines.

3.1 Writing Guidelines

It is best to think of these writing projects as officially assigned papers in which you completely explain and justify your analyses of the problems. You may work with others in solving these problems but there is to be **no collaboration on the written exposition of the solutions.** In addition I expect your papers to be

- Fully documented – specifically:

  1. You must include a reference paragraph at the beginning of your paper either affirming the work is completely yours or listing each resource you use: names of participants in discussions (other than the in-class discussions), technological tools, reference texts employed, and anything else other than your own thoughts.
  2. Any idea obtained during brainstorm sessions or in discussions is cited in-line.
  3. All textbook results (theorems, propositions, and lemmas) are cited in-line and include the name of the result.
  4. Any use of technology is cited in-line.

- Carefully handwritten in ink or written with a word processor. (I can show you how to use Scientific Notebook in the labs or you can use Microsoft Word. Please check with me before using any other program.)
- Written using complete, accurately punctuated sentences.
- Presented in active voice, the first person plural and with a clear, easy-to-follow expository style.
- Targeted at an audience consisting of students not in this class but with an equivalent mathematical background – say those currently in another section of this course.