

November 22, 2006

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Name

Directions: Be sure to include in-line citations, including page numbers if appropriate, every time you use the results of discussion, a text, notes, or technology. **Only write on one side of each page.**

*"'Know thyself?' If I knew myself, I'd run away."* – Johann von Goethe

### Problems

1. Prove the icosahedral group has no subgroup of order 30.
2. Prove no group of order  $p^l$ , where  $p$  is prime and  $l > 1$ , is simple.
3. List all subgroups of the dihedral group  $D_4$ , and divide them into conjugacy classes.
4. Do **both** of the following.
  - (a) Let  $H$  be a normal subgroup of a group  $G$  of order  $|H| = 2$ . Prove  $H$  is in the center of  $G$ .
  - (b) Let  $H$  be a normal subgroup of prime order  $p$  in a finite group  $G$ . Suppose  $p$  is the smallest prime dividing  $|G|$ . Prove  $H$  is in the center of  $G$ .
5. Prove no group of order  $p^2q$ , where  $p$  and  $q$  are distinct primes, is simple.
6. Do **one** of the following.
  - (a) Prove the only simple groups of order less than 60 are groups of prime order.
  - (b) Classify all groups of order 33.
  - (c) Classify all groups of order 18.