

Not Turned In

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Name

Be sure to re-read the **WRITING GUIDELINES rubric**, since it defines how your project will be graded. In particular, you may discuss this project with others but **you may not collaborate on the written exposition of the solution**.

*“Do not imagine that Mathematics is hard and crabbed, and repulsive to common sense. It is merely the etherealization of common sense.”* – Lord Kelvin

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### Project Problem

Do **both** of the following.

1. If  $A$  is  $m \times n$  and  $B$  is  $n \times m$  (note that neither  $A$  nor  $B$  is square and hence neither has a determinant), show that the determinant of the partitioned matrix,  $C$ , below satisfies  $\det(C) = \det(AB)$ .

$$C = \begin{bmatrix} O & A \\ -B & I \end{bmatrix}$$

[Hint: Consider multiplying by  $\begin{bmatrix} I & O \\ B & I \end{bmatrix}$ .]

Do an example with  $m < n$  and an example with  $m > n$ . Do you understand why the second example has  $\det(AB) = 0$ ?

2. Explain why the following  $5 \times 5$  matrix that has a  $3 \times 3$  zero submatrix is definitely singular (regardless of the 16 non-zeros marked by  $x$ 's.)

$$\begin{bmatrix} x & x & x & x & x \\ x & x & x & x & x \\ 0 & 0 & 0 & x & x \\ 0 & 0 & 0 & x & x \\ 0 & 0 & 0 & x & x \end{bmatrix}$$