Math 290 Fall 2006 Project 09

Due November 3

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Name

Be sure to re-read the WRITING GUIDELINES rubric, since it defines how your project will be graded. In particular, you may discuss this project with others but **you may not collaborate on the written exposition of the solution.**

"The road to wisdom? Well its plain and simple to express: Err and err and err again, but less and less and less." -Piet Hein, poet and scientist (1905-1996)

Project Problem

Suppose $B = \left\{\vec{b}_1, \vec{b}_2, \vec{b}_{3,\cdots}, \vec{b}_{p-1}, \vec{b}_p, \vec{b}_{p+1}, \vec{b}_{p+2}, \cdots, \vec{b}_m\right\}$ is an orthonormal basis for $\mathbf{C}^m$ and let
$V = \left\langle\left\{\vec{b}_1, \vec{b}_2, \vec{b}_{3,\cdots}, \vec{b}_{p-1}, \vec{b}_p\right\}\right\rangle$ be the subspace of $\mathbf{C}^m$ spanned by the first p vectors in B.
Define $V^\perp = \{\vec{w} \in \mathbf{C}^m : \ \langle \vec{w}, \vec{v}\rangle \ = 0$ **for each and every** $\vec{v} \in V\ \}$. The set $V^\perp$ is called the **orthogonal complement** of V **in** $\mathbf{C}^m$.
Make sure you understand the definition of $V^\perp$ before proceding.

1. Show that $V^\perp$ is a subspace of $\mathbf{C}^m$.
2. Show that $V^\perp = \left\langle\left\{\vec{b}_{p+1}, \vec{b}_{p+2}, \cdots, \vec{b}_m\right\}\right\rangle$
3. Show that $\mathbf{C}^m = V + V^\perp = \left\{\vec{v} + \vec{w} : \vec{v} \in V \ \text{ and } \vec{w} \in V^\perp\right\}$.