Math 290 Fall 2006 Project 08

DO NOT turn in

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Name

Be sure to re-read the WRITING GUIDELINES rubric, since it defines how your project will be graded. In particular, you may discuss this project with others but **you may not collaborate on the written exposition of the solution.**

"Anyone who cannot cope with mathematics is not fully human. At best he is a tolerable subhuman who has learned to wear shoes, bathe, and not make messes in the house." – Robert Heinlein in *Time Enough for Love.*

Preliminary Information

The following questions refer to **Example VSIS,** The vector space, $\mathbf{C}^\infty = \{(c_0, c_1, c_2, c_3, ...) : c_i \in \mathbf{C},\ i \in \mathbf{N}\}$, of infinite sequences. Recall that in this vector space equality, addition and scalar multiplication are given by:

- Equality: $(c_0, c_1, c_2, ...) = (d_0, d_1, d_2, ...)$ if and only if $c_i = d_i$ for all $i \geq 0$
- Vector Addition: $(c_0, c_1, c_2, ...) + (d_0, d_1, d_2, ...) = (c_0 + d_0, c_1 + d_1, c_2 + d_2, ...)$
- Scalar Multiplication: $\alpha(c_0, c_1, c_2, c_3, ...) = (\alpha c_0, \alpha c_1, \alpha c_2, \alpha c_3, ...)$

Recall also that the set $A = \{(a, a+k, a+2k, a+3k, \cdots) : a, k \in \mathbf{C}\}$ of **arithmetic sequences** is a subspace of $\mathbf{C}^\infty$.

Problems

1. Do both of the following:

   (a) Use the methods of this section of the textbook to Determine whether or not the the set $S = \{(3, -6, 12, -24, 48, \cdots), (5, 10, 20, 40, 80, \cdots,), (2, 0, 8, 0, 32, 0, 128, \cdots)\}$ is linearly independent or linearly dependent in $\mathbf{C}^\infty$.

   (b) If the set is linearly dependent, exhibit a nontrivial relation of linear dependence.

2. Use the definition of $A = \{(a, a+k, a+2k, a+3k, \cdots) : a, k \in \mathbf{C}\}$ and the methods of this section to find a set, T, of vectors that span A and that are also linearly independent.

3. Determine whether or not the subset $W = \left\{\left(a, ar, ar^2, ar^3, ar^4, \cdots\right) : a, r \in \mathbf{C}\right\}$ of **geometric sequences** is a subspace of $\mathbf{C}^\infty$.