Due October 20

                                                        Name

Be sure to re-read the WRITING GUIDELINES rubric, since it defines how your project will be graded. In particular, you may discuss this project with others but **you may not collaborate on the written exposition of the solution.**

"It is hard to know what you are talking about in mathematics, yet no one questions the validity of what you say. There is no other realm of discourse half so queer." – J. R. Newman

Do any two (2) of the following.

1. Given a square matrix A, we (inductively) define $A^1 = A$ and $A^{n+1} = A(A^n)$ for $n = 1, 2, \cdots$.

   Claim: If A is a square matrix with the property that $N(A^2) = N(A^3)$ then $N(A^3) = N(A^4)$. If this claim is true, provide a proof. If it is false, provide an example of a matrix A for which the claim fails.

2. Suppose we know that a certain 5×5 matrix A can be written as

$$A = BC$$

   where B is 5×4 and C is 4×5. Prove that A is not invertible.

3. Do both of the following:

   (a) Find, with explanation, a matrix B where the null space of B is the column space of $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$.

   (b) Find, with explanation, a matrix A where the column space of A is the null space of $B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$.