Math 290 **Fall 2006** **Project 04**

Due September 29

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Name

Be sure to re-read the WRITING GUIDELINES rubric, since it defines how your project will be graded. In particular, you may discuss this project with others but **you may not collaborate on the written exposition of the solution.**

"Obvious" is the most dangerous word in mathematics." – Eric Temple Bell

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Linear Independence

Do One (1) **of the following.**

1. Theorem DLDS (Dependency in Linearly Dependent Sets) says: Suppose that $S = \{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, ..., \mathbf{u}_n\}$ is a set of vectors. Then S is a linearly dependent set if and only if there is an index t, $1 \leq t \leq n$ such that $\mathbf{u}_t$ is a linear combination of the vectors $\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, ..., \mathbf{u}_{t-1}, \mathbf{u}_{t+1}, ..., \mathbf{u}_n$.

   Thus we know that in any linearly independent set there must be a vector that can be written as a linear combination of the other vectors of that set. The purpose of this first problem is to refine that claim slightly. Specifically, using any results in the book up through Chapter V - Section LDS, prove the following.

   Theorem 1 *DLDSPV (Dependency in Linearly Dependent Sets, Previous Vectors) Suppose that $S = \{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, ..., \mathbf{u}_n\}$ is a set of vectors. Then S is a linearly dependent set if and only if there is an index t, $1 \leq t \leq n$ such that u_t is a linear combination of the vectors $\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, ..., \mathbf{u}_{t-1}$ which have subscripts smaller than t.*

2. Suppose that $S = \{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, ..., \mathbf{u}_n\}$ and $T = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, ..., \mathbf{v}_p\}$ are two linearly independent sets of vectors in $\mathbf{C}^m$. Suppose further that the only vector common to the span of S and the span of T is the zero vector. That is, $< S > \cap < T > = \{\mathbf{0}\}$. Prove, using any results in the book through Chapter V - Section LDS, that the set $W = S \cup T = \{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, ..., \mathbf{u}_n, \mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, ..., \mathbf{v}_p\}$ is also linearly independent.