Semester Review

The Big Picture:

Chapter 5: Presents the basics of the theory of integration

Chapter 6: Standard applications of definite integrals

Chapter 7: How to find antiderivatives

- (to exploit the fundamental theorem for computing definite integrals)

Chapter 8: Sequences and Series

- Discrete analogs to functions and antiderivatives.
- How to determine convergence.
- Every power series is a function with a special domain.
- Some functions are equal to power series (their Taylor Series).

The Medium Picture:

Chapter 5

The basic theory of integration

- Discrete Domain Functions
  1. Sequences
  2. Derivatives
  3. Antiderivatives
  4. Finite Sums
  5. Fundamental Theorems of Sequences
- Interval Domain Functions
  1. Antiderivatives
  2. Riemann Sums and definite integrals
  3. Fundamental Theorems of Calculus
  4. Basic Substitution techniques
  5. Beginning Differential Equations
  6. Numerical Approximation

Chapter 6

Standard Applications of Definite Integrals

- 1. Areas between curves
  2. Volumes of solids
  3. Arc length of curves
  4. Surface areas of surfaces of revolution
  5. (Not covered in class) Work, Fluid force

Chapter 7

Methods of Integration

- 1. Intermediate Substitution techniques
  2. Tables of integrals
  3. Integration by parts
  4. Trigonometric methods
  5. Partial Fractions
  6. (Not covered in class) First Order linear Differential Equations
  7. Improper integrals
  8. Hyperbolic functions

Chapter 8

Infinite sequences and series

- 1. Sequences, their limits, convergence
     (a) Linearity
  2. Infinite Series = Improper Summations
     (a) Linearity
  3. Summable Series
     (a) Geometric Series $\sum r^k$
     (b) $\sum \frac{1}{k^p}$
  4. Tests for convergence
     (a) P-Series
     (b) Divergence
     (c) Integral
     (d) Comparison (direct and limit)
     (e) Ratio and Root
     (f) Alternating Series
  5. Absolute and Conditional convergence
  6. Power series
  7. Taylor Series and Maclaurin Series

More Detailed Outline

Chapter 5: The fundamentals of integration

Discrete Domain Functions

- Sequences

  1. A function with a discrete domain.

- Derivatives

$$D_k\left[a\left(k\right)\right] = \frac{a\left(k+1\right) - a\left(k\right)}{1}$$

  1. Analogous to

$$f\,'\left(x\right) = \lim_{\Delta x \to 0} \frac{f\left(x+\Delta x\right) - f\left(x\right)}{\Delta x}$$

  2. Geometric Meaning: Slope of line segment joining points on graph.
  3. Derivative Rules
     (a) $D_k\left[k^{\underline{p}}\right] = pk^{\underline{p-1}}$
     (b) $D_k\left[r^k\right] = \left(r-1\right)r^k$
     (c) Linearity
     (d) Product Rule

- Antiderivatives

  1. Indefinite Summation $\sum a\left(k\right) = A\left(k\right) + C$
  2. Antiderivative Formulas
     (a) $D_k\left[k^{\underline{-p}}\right] = -p\left(k+1\right)^{\underline{-p-1}}$
     (b) $D_k\left[r^k\right] = \left(r-1\right)r^k$

- Finite Sums = Definite Summation

  1. $\sum_{k=1}^{n} a\left(k\right) = a\left(1\right) + a\left(2\right) + \cdots + a\left(n\right)$

- Fundamental Theorems of Sequences

  1. Every sequence has a discrete antiderivative (2nd Fundamental Theorem)

$$D_k\left[\sum_{j=m}^{k-1} a\left(j\right)\right] = a\left(k\right)$$

  2. Summing the terms of a sequence $a\left(k\right)$ can be shortened (1st Fundamental Theorem) provided one can find a discrete antiderivative of $a\left(k\right)$.

$$\sum_{k=m}^{n} a\left(k\right) = A\left(k\right)\Big|_{m}^{n+1} = A\left(n+1\right) - A\left(m\right)$$

Interval Domain Functions

- Antidifferentiation:

  1. Reversing the process of taking derivatives.

- Riemann Sums and definite integrals:

$$\sum_{k=1}^{n} f\left(x_k^*\right) \Delta x_k$$

  1. Using sums of linear approximations over small intervals to approximate effects of functions over large intervals.
  2. A Riemann Sum depends on
     (a) the function $f(x)$
     (b) an interval $[a, b]$ in the domain of f
     (c) a partition $P: \; a = x_0 < x_1 < \cdots < x_n = b$ of the interval
     (d) a selection of points $x_1^*, \; x_2^*, \cdots, \; x_n^*$ where x_k^* is a point in the k'th subinterval $[x_{k-1}, x_k]$ of the partition.
  3. A definite integral is the limit as the partition norm goes to 0 of all possible Riemann sums for a function f on the interval $[a, b]$

$$\int_a^b f(x) \; dx = \lim_{||P \to 0||} \sum_{k=1}^{n} f\left(x_k^*\right) \Delta x_k$$

- The **Fundamental Theorems of Calculus**

  1. Second Fundamental Theorem. Every continuous function has an antiderivative. (Actually infinitely many)

$$\frac{d}{dx} \int_a^x f(t) \; dt = f(x)$$

  2. First Fundamental Theorem. Computation of definite integrals (limits of Riemann Sums) can be shortened by the use of antiderivatives (provided one can find an antiderivative for f.)

$$\int_a^b f(x) \; dx = F(b) - F(a)$$

- Basic Integration techniques

  1. Substitution
  2. Rule of Thumb usually works for simple integrals

- Differential Equations:

  1. **Not Covered** Graphical solutions:
     (a) Slope fields (direction fields)
     (b) The program Differential Systems on the university Macintoshes

2. **Not Covered** Numerical solutions:
   (a) Euler's Method
   (b) The numerical formulas arising from using linear approximation on slope fields.
3. Symbolic solutions
   (a) Separation of variables
4. Basic situations using differential equations:
   (a) Exponential models
   (b) Carbon dating
   (c) Orthogonal trajectories
   (d) fluid flow through an orifice

- Mean Value Theorem for Integrals and **Average Value** of a continuous function

  1. Average of f on $[a, b]$ is
  $$\frac{1}{b-a}\int_a^b f(x)\ dx$$
  2. Geometric meaning of the average value: height of rectangle over base $a \le x \le b$ with same area as $\int_a^b f(x)\ dx$.

- Numerical Integration (Approximating definite integrals with attention to accuracy)

  1. Left Endpoint Rule
  2. Right Endpoint Rule
  3. Trapezoid Rule:
     (a) $T_n = \frac{1}{2}(L_n + R_n)$
     (b) Error Bound: $|I - T_n| \le \frac{(b-a)^3}{12n^2} M$
  4. Midpoint Rule
  5. Simpson's Rule:
     (a) $S_n = \frac{1}{3}(T_n + 2M_n)$
     (b) Error Bound $|I - S_n| \le \frac{(b-a)^5}{180n^4} K$

Chapter 6: Applications of definite integrals

- Area between curves

- Volumes of solids

  1. Cross-sectional areas
  $$V = \int_a^b A(x)\ dx$$

(a) Disks
(b) Washers

2. Cylindrical Shells

$$V = 2\pi \int_a^b (\text{radius})(\text{height})\, dx$$

- Arc length and Surface area:

$$\begin{aligned} ds &= \sqrt{1 + [f'(x)]^2} dx \\ S &= \int_a^b ds \\ SA &= \int_a^b 2\pi f(x)\; ds \end{aligned}$$

1. Many problems are 'cooked' so that the algebra simplifies to remove the square root.

- Physical Applications

1. Work done by a variable force

$$W = \int_a^b F(x)\; dx$$

(a) Hooke's Law
(b) **Not Covered** Work done in pumping out a tank
   i. Riemann Sum of form $\sum \Delta W$ where

$$\Delta W = \left(\Delta V \,\text{m}^3\right)\left(\rho\, \frac{\text{N}}{\text{m}^3}\right)(\Delta y\, \text{m})$$

2. **Not Covered** Total fluid force on a vertical surface

$$F = \int_a^b \left(\rho\, \frac{\text{lb}}{\text{ft}^3}\right)(h(x)\;\; \text{ft})(L(x)\;\; \text{ft})\;\; dx\,\text{ft}$$

Chapter 7: Methods of Integration

- Basic substitution:

1. rule of thumb
2. algebra first – then rule of thumb
   (a) complete the square
3. substitution for a u for which the du already is in the problem – then use algebra to simplify – then make a rule of thumb substitution
4. fractional exponents

- Use of tables

- Integration by Parts

$$\int u\,dv = uv - \int v\,du$$

  1. When to use.
  2. How it occurs in definite integrals

- Trigonometric Methods

  1. Powers of Sine and Cosine
     (a) Look for an odd power of either $\sin(x)$ or $\cos(x)$
        i. substitute u for the other one (e.g. if $\cos(x)$ occurs to an odd power, let $u = \sin(x)$ so that $du = \cos(x)\ dx$)
        ii. Use trigonometric identites to swap out even powers of the non- u trig function.
     (b) If both $\sin(x)$ and $\cos(x)$ are to even powers
        i. Use the half-angle trigonometric identies to reduce to an odd power

$$\begin{aligned}\sin^2(x) &= \frac{1}{2}(1-\cos(2x))\\ \cos^2(x) &= \frac{1}{2}(1+\cos(2x))\\ \sin(2x) &= 2\sin(x)\cos(x)\end{aligned}$$

  2. Powers of Secant and Tangent (or Cosecant and Cotangent)
     (a) Look for an even power of the secant
        i. substitute for $u = \tan(x)$ so $du = \sec^2(x)\ dx$
        ii. Use trigonometric identites to swap extra even powers of secant for even powers of tangent.
     (b) Look for an odd power of the tangent
        i. substiture $u = \sec(x)$ so $du = \sec(x)\tan(x)\ dx$
        ii. Use trigonometric identites to swap extra even powers of tangent for even powers of secant

- Trigonometric substitutions

  1. If $a^2 - u^2$ occurs, try $u = \sin(x)$ or $u = \tanh(x)$
  2. If $a^2 + u^2$ occurs, try $u = \tan(x)$ or $u = \sinh(x)$
  3. If $u^2 - a^2$ occurs, try $u = \sec(x)$ or $u = \cosh(x)$

- Partial Fractions

  1. Only works on **proper** fractions so **divide first.**
  2. decompose into sums of fractions with linear, irreducible quadratic, or powers of linear or irreducible quadratic denominators
  3. Integrate each of the simpler fractions using other techniques

- **Not Covered** First order **Linear** differential equations

  1. Compute the integrating factor for the DE

$$\begin{aligned} \frac{dy}{dx} + P(x)\, y &= Q(x) \\ \text{Int.Factor } I &= e^{\int P(x) dx} \end{aligned}$$

  2. Multiply both sides of the differential equation above by the integrating factor so the left hand side turns into

$$\frac{d}{dx}\left[I\ y\right]$$

  3. Solve by integrating both sides.

- Improper integrals

  1. Can only compute improper integrals with **one** impropriety
  2. Types

$$\int_a^\infty f(x)\ dx$$
$$\int_{-\infty}^b f(x)\ dx$$
$$\int_{-\infty}^\infty f(x)\ dx$$

$$\begin{aligned} \int_a^b f(x)\ dx \text{ where } x &= b \text{ is a vertical asymptote} \\ \int_a^b f(x)\ dx \text{ where } x &= a \text{ is a vertical asymptote} \\ \int_a^b f(x)\ dx \text{ where } x &= c \text{ is a vertical asymptote } \text{ and } a < c < b \end{aligned}$$

  3. Methodology is exactly the same as computing whether or not an infinite series converges.

- Hyperbolic Tirgonometric functions

  1.

$$\begin{aligned} \sinh(x) &= \frac{1}{2}\left(e^x - e^{-x}\right) \\ \cosh(x) &= \frac{1}{2}\left(e^x + e^{-x}\right) \\ \tanh(x) &= \frac{\sinh(x)}{\cosh(x)}, \quad \text{etc.} \\ \cosh^2(x) - \sinh^2(x) &= 1 \end{aligned}$$

  2.

$$\begin{aligned} \frac{d}{dx}\left[\sinh(x)\right] &= \cosh(x) \\ \frac{d}{dx}\left[\cosh(x)\right] &= \sinh(x) \end{aligned}$$

Chapter 8: Sequences and Series

- Deduce the general term from a given sequence written in 'dot, dot, dot' form.
- The definition of what it means for a sequence a_n to converge

   1. $\lim_{n\to\infty} a_n = L$ means:
      Given any positive number ε, there is a number N for which whenever $n > N$ we have

$$L - \varepsilon < a_n < L + \varepsilon$$

- Sequences have discrete derivatives and discrete antiderivatives analogous to derivatives and antiderivatives of continuous functions.

   1. Think of $\Delta k = 1$

$$\tfrac{d}{dx} f(x) = \lim_{\Delta x \to 0} \tfrac{f(x+\Delta x) - f(x)}{\Delta x} \qquad D_k[a(k)] = \tfrac{a(k+1) - a(k)}{1}$$

$$F'(x) = f(x) \qquad D_k[A(k)] = a(k)$$

$$\textstyle\int_a^b f(x)\, dx = F(x)\Big|_a^b \qquad \sum_{k=1}^{n} a(k) = A(k)\Big|_1^{n+1}$$

- Infinite Series are the discrete analogs of improper integrals of continuous functions.

$$\textstyle\int_a^\infty f(x)\, dx = \lim_{b\to\infty} \int_a^b f(x)\, dx = \lim_{b\to\infty} F(x)\Big|_a^b \qquad \sum_{k=1}^{\infty} a(k) = \lim_{n\to\infty} \sum_{k=1}^{n} a(k) = \lim_{n\to\infty}$$

- The bounded, monotonic convergence theorem (BMCT) for sequences.

   1. A sequence a_n is bounded above if there is a number M for which $a_n \le M$ for all n.
   2. A sequence a_n is bounded below if there is a number m for which $m \le a_n$ for all n.
   3. Sequences can be monotone in four ways: increasing, decreasing, nondecreasing, nonincreasing.

- Textbook Notation for infinite series $\sum_{k=1}^{\infty} a_k$.

   1. Let A_k be any discrete antiderivative of a_k. (One choice is S_k where $S_1 = 0, S_2 = a_1, S_3 = a_1 + a_2, \cdots, S_n = \sum_{k=1}^{n-1} a_k$
   2. Then, the infinite series $\sum_{k=1}^{\infty} a_k$ converges if and only if the sequence of partial sums $S_n = \sum_{k=1}^{n-1} a_k$ converges which is true if and only if $\lim_{n\to\infty} S_n$ exists.

- Useful series

   1. Geometric Series converges only when $|r| < 1$

$$\sum_{k=0}^{\infty} ar^k$$

   2. $\sum_{k=1}^{\infty} 1/k^{\underline{n}}$ can be summed exactly by using discrete antiderivatives.
   3. Telescoping series can be summed by 'telescoping' the partial sums.

4. p- series which converge if and only if $p > 1$(but we don't know how to find the sum)

$$\sum_{k=1}^{n} \frac{1}{k^p}.$$

- Linearity of **convergent** series
  1. If $\sum_{k=1}^{\infty} a(k)$ and $\sum_{k=1}^{\infty} b(k)$ both converge then so does
     (a) $\sum_{k=1}^{\infty} [r\, a(k) + s\, b(k)]$ where r and s are any constants.

- If r and s are constants – neither equal to 0 then
  1. If any two of $\sum_{k=1}^{\infty} a(k)$, $\sum_{k=1}^{\infty} b(k)$,and $\sum_{k=1}^{\infty} [r\, a(k) + s\, b(k)]$ converge, then so does the third.

- Sums involving **divergent** series
  1. If $\sum_{k=1}^{\infty} a(k)$ converges and $\sum_{k=1}^{\infty} b(k)$ diverges then
     – $\sum_{k=1}^{\infty} [r\, a(k) + s\, b(k)]$ diverges as long as $s \neq 0$.

Tests for Convergence of $\sum_{k}^{\infty} a_k$

- Geometric Series can be summed exactly
- p - series test
- Divergence test

$$\lim_{k\to\infty} a_k = \text{anything but } 0$$

  1. Can be applied to any series
  2. Can only inform that a series diverges – can never inform that a series converges

- Integral Test

$$\sum_{k=1}^{\infty} f(k) \quad \text{and} \quad \int_{1}^{\infty} f(x)\, dx \;\; \text{converge or diverge together}$$

  1. Applies only for a positive, decreasing continuous function f

- Direct Comparison Test
  1. Applies only to series consisting of nonnegative terms
  2. If $\sum_{k}^{\infty} c_k$ dominates $\sum_{k}^{\infty} a_k$ and converges, then so does $\sum_{k}^{\infty} a_k$
  3. $\sum_{k}^{\infty} c_k$ is dominated by $\sum_{k}^{\infty} a_k$ and diverges, then so does $\sum_{k}^{\infty} a_k$

- Limit Comparison Test

1. Applies only to series consisting of positive terms
2. If $\lim_{k\to\infty} \frac{a_k}{b_k} = L$
   (a) L finite and non-zero, then $\sum_k^\infty a_k$ and $\sum_k^\infty b_k$ converge or diverge together.
   (b) $L = 0$ and $\sum_k^\infty b_k$ converges then $\sum_k^\infty a_k$ converges
   (c) $L = \infty$ and $\sum_k^\infty b_k$ diverges then $\sum_k^\infty a_k$ diverges

- Ratio Test and Root Test

1. Applies only to series with positive terms
2. If $\lim_{k\to\infty} \frac{a_{k+1}}{a_k} = L$ or $\lim_{k\to\infty} \sqrt[k]{a_k} = L$ where
   (a) $L < 1$ then $\sum_k^\infty a_k$ converges.
   (b) $L > 1$ then $\sum_k^\infty a_k$ diverges
   (c) $L = 1$ then no information

- Alternating Series Test

1. If $p_k > 0$ with
   (a) p_k a decreasing sequence
   (b) $\lim_{k\to\infty} p_k = 0$

   Then $\sum_k^\infty a_k = \sum_k^\infty (-1)^k p_k$ converges.
2. Easy to approximate:
   (a) If $\sum_{k=1}^\infty (-1)^k a_k$ converges to S, then $\left| S - \sum_{k=1}^n (-1)^k a_k \right| < a_{n+1}$

Absolute and Conditional Convergence

- If $\sum_k^\infty |a_k|$ converges then so does $\sum_k^\infty a_k$ and the latter's convergence is absolute.

1. Rearrangements of absolutely convergent series do not affect either the fact of convergence or the sum.

- If $\sum_k^\infty |a_k|$ diverges and $\sum_k^\infty a_k$ converges then the latter's convergence is conditional.

1. A conditionally convergent series may be rearranged to converge to any number or to diverge to either plus or minus infinity.

Power Series

- Any series in either of the forms

$$\begin{aligned} f(x) &= \sum_{k}^{\infty} a_k x^k \\ f(x) &= \sum_{k}^{\infty} a_k (x-c)^k \end{aligned}$$

- Any power series is a function and converges on one of the following sets (which is the domain of the function.)

  1. At only one point
  2. On a finite interval centered at the number $x = c$
  3. On the entire real line.

- Use Generalized Ratio or Root Tests (Apply the standard tests to the absolute value series) to detect the radius of convergence.

- Check the endpoints separately

- Power series can be differentiated and integrated term-by-term.

  1. After integrating or differentiating, the resulting series have the same Radius Of Convergence as the original series.
  2. After integrating or differentiating, the endpoints can behave differently than in the original.

Taylor Series and Maclaurin Series

- Every infinitely differentiable function $f(x)$ gives rise to a power series.

$$\sum_{k=0}^{\infty} \frac{1}{k!} f^{(k)}(c)\ (x-c)^k \quad \text{(Taylor Series)}$$

$$\sum_{k=0}^{\infty} \frac{1}{k!} f^{(k)}(0)\,(x-0)^k \quad \text{(Maclaurin Series)}$$

- A Maclaurin series $\sum_{k=0}^{\infty} \frac{1}{k!} f^{(k)}(0)\ x^k$ has the same outputs as the function $f(x)$ if and only if

$$\lim_{n \to \infty} R_n(x) = 0$$

  where M denotes the absolute maximum of $\left[f^{(n+1)}(x)\right]$ and

$$|R_n(x)| \leq \frac{M}{(n+1)!} |x|^{n+1}$$

- A few known functions and the Taylor Series they equal include:

$$\begin{aligned}
\frac{1}{1-x} &= \sum_{k=0}^{\infty} x^k, \quad -1 < x < 1 \\
e^x &= \sum_{k=0}^{\infty} \frac{x^k}{k!}, \quad \text{for all } x \\
\cos(x) &= \sum_{k=0}^{\infty} (-1)^k \frac{x^{2k}}{(2k)!}, \quad \text{for all } x \\
\sin(x) &= \sum_{k=0}^{\infty} (-1)^k \frac{x^{2k+1}}{(2k+1)!}, \quad \text{for all } x \\
(1+x)^p &= 1 + px + \frac{p(p-1)}{2!}x^2 + \frac{p(p-1)(p-2)}{3!}x^3 + \cdots
\end{aligned}$$

The last is the binomial series and converges:

1. (a) For all x if p is an **integer** that is positive.
   (b) For $-1 < x < 1$ if $p \leq -1$
   (c) For $-1 \leq x \leq 1$ if $p > 0$ but p is **not** an integer.
   (d) For $-1 < x \leq 1$ if $-1 < p < 0$.

- The Taylor series for many other functions can be computed 'easily' by noting that those functions are combinations of the above or the derivatives or integrals of the above.

1. **Example:**

$$\begin{aligned}
\frac{1}{1-x} &= \sum_{k=0}^{\infty} x^k, \quad -1 < x < 1 \\
\frac{1}{1+x^2} &= \sum_{k=0}^{\infty} \left(-x^2\right)^k, \quad -1 < x < 1 \\
&= \sum_{k=0}^{\infty} (-1)^k x^{2k}, \quad -1 < x < 1 \\
\arctan(x) &= \int \frac{1}{1+x^2}\,dx \\
&= \int \sum_{k=0}^{\infty} (-1)^k x^{2k} \; dx \\
&= \sum_{k=0}^{\infty} \int (-1)^k x^{2k}\,dx \\
&= \sum_{k=0}^{\infty} (-1)^k \frac{x^{2k+1}}{2k+1} \quad -1 \leq x \leq 1
\end{aligned}$$

Analogies between Sequences/Series and Functions/Integrals

$D_k\left[k^{\underline{p}}\right] = pk^{\underline{p-1}}$		$\frac{d}{dx}\left[x^n\right] = nx^{n-1}$
$D_k\left[k^{\underline{-p}}\right] = -p\left(k+1\right)^{\underline{-p-1}}$		$\frac{d}{dx}\left[x^{-n}\right] = -nx^{-n-1}$
$D_k\left[r^k\right] = \left(r-1\right)r^k$		$\frac{d}{dx}\left[c^x\right] = \ln\left(c\right)c^x$
$D_k\left[A\left(k\right)\right] = a\left(k\right) \rightarrow \sum a\left(k\right) = A\left(k\right) + C$		$\frac{d}{dx}\left[F\left(x\right)\right] = f\left(x\right) \rightarrow \int f\left(x\right)dx = F\left(x\right) + C$
$\sum k^{\underline{p}} = \frac{1}{p+1}k^{\underline{p+1}} + C$		$\int x^n dx = \frac{1}{n+1}x^{n+1} + C$
$\sum k^{\underline{-p}} = \frac{1}{-p+1}\left(k-1\right)^{\underline{-p+1}} + C$, if $p \neq 1$		$\int x^{-n}dx = \frac{1}{-n+1}x^{-n+1} + C$, if $n \neq 1$
$\sum \frac{1}{k^{\underline{1}}} = H\left(k\right) + C$ Harmonic Series		$\int \frac{1}{x}dx = \ln\lvert x\rvert + C$
$\sum r^k = \frac{1}{r-1}r^k + C,\ \ r \neq 1$		$\int r^x dx = \frac{1}{\ln(r)}r^x + C,\ \ r \neq 1$
$\sum 1^k = k + C$		$\int 1dx = x + C$
$\sum_{k=m}^{n} a\left(k\right) = A\left(k\right)\rvert_m^{n+1} = A\left(n+1\right) - A\left(m\right)$	1 FT	$\int_a^b f\left(x\right)dx = F\left(x\right)\Big\rvert_a^b = F\left(b\right) - F\left(a\right)$
$D_k\left[\sum_{j=m}^{k-1} a\left(j\right)\right] = a\left(k\right)$	2 FT	$\frac{d}{dx}\int_a^x f\left(t\right)\ dt = f\left(x\right)$
$D_k\left[u_k v_k\right] = u_k D_k\left[v_k\right] + v_{k+1}D_k\left[u_k\right]$		$\frac{d}{dx}\left[uv\right] = u\frac{dv}{dx} + v\frac{du}{dx}$
$\sum_{k=0}^{n} U_k\, v_k = U_k V_k\rvert_0^{n+1} - \sum_{k=0}^{n} V_{k+1}u_k$		$\int_a^b u\,dv = uv\Big\rvert_a^b - \int_a^b v\,du$
$\sum_{k=m}^{\infty} a\left(k\right) = \lim_{n\to\infty}\sum_{k=m}^{n} a\left(k\right)$		$\int_a^{\infty} f\left(x\right)dx = \lim_{b\to\infty}\int_a^b f\left(x\right)dx$
$0 \leq a\left(k\right) \leq b\left(k\right)$ and $\sum_{k=m}^{\infty} b\left(k\right)$ conv. $\Longrightarrow \sum_{k=m}^{\infty} a\left(k\right)$ conv.		$0 \leq f\left(x\right) \leq g\left(x\right)$ and $\int_a^{\infty} g\left(x\right)dx$ conv. $\Longrightarrow \int_a^{\infty} f\left(x\right)dx$ conv.
$0 \leq a\left(k\right) \leq b\left(k\right)$ and $\sum_{k=m}^{\infty} a\left(k\right)$ div. $\Longrightarrow \sum_{k=m}^{\infty} b\left(k\right)$ div.		$0 \leq f\left(x\right) \leq g\left(x\right)$ and $\int_a^{\infty} f\left(x\right)dx$ div. $\Longrightarrow \int_a^{\infty} g\left(x\right)dx$ div.
$\lim_{n\to\infty} a_n \neq 0 \Longrightarrow \sum_{k=1}^{\infty} a_k$ diverges		$\lim_{x\to\infty} f\left(x\right) = c \neq 0 \Longrightarrow \int_1^{\infty} f\left(x\right)\ dx$ div.
Fns as series ($f\left(x\right) = \sum_{k=1}^{\infty} a_k x^k$)		Fns as integrals ($\Gamma\left(x\right) = \int_0^{\infty} t^{x-1}e^{-t}\,dt$)
$\sum_{k=0}^{\infty} \frac{1}{k!}f^{(k)}\left(c\right)\ \left(x-c\right)^k$ (Taylor Series)		