Mathematics 122A,B Fall 2004 Project 1

September 7, 2004

Study Group Members	**Name**

Only write on one side of each page.

I encourage you to work with others on this quiz. As with all writing you should work out the details in a draft before writing a final solution. Be sure to follow the writing guidelines listed in the course information sheet unless explicitly directed to do otherwise in the problem statement. You do not need to include every algebra or arithmetic step but you should include enough detail to allow a member of your target audience to reconstruct any missing steps. Be sure to include in-line citations, with page numbers if appropriate, every time you use the results of discussion, a text, notes, or technology. If you include graphs, they should be done carefully on graph paper. Finally, there is to be no collaboration in the writing of your solution even if you worked out the details with other people.

"*Personally, I'm always ready to learn, although I do not always like being taught.*" – Winston Churchill

Problems

Do **one** of the following. The first problem points out some basic properties of a particularly interesting subset of the Real numbers and the second develops the 'Product Rule' for discrete derivatives.

1. "Cantor's Dust" is a subset C of the real numbers obtained in the following fashion. Let $C_1 = \{x \in \mathbf{R} : 0 \le x \le 1\} = [0,1]$. Form C_2 by removing the (open) middle one-third of C_1.Specifically, $C_2 = \left[0, \frac{1}{3}\right] \cup \left[\frac{2}{3}, 1\right]$. Form $C_3$by removing the open middle one-third of each of the intervals in C_2 so that $C_3 = \left[0, \frac{1}{9}\right] \cup \left[\frac{2}{9}, \frac{1}{3}\right] \cup \left[\frac{2}{3}, \frac{7}{9}\right] \cup \left[\frac{8}{9}, 1\right]$. Continue in this fashion forming the infinite sequence of sets $C_1, C_2, C_3, \cdots$. Cantor's Dust is the set C obtained as the limit of this process. It is a fact (usually proven using techniques from advanced calculus and topology) that there are an uncountably infinite number of points in the set C.
   (a) Give a reasonable argument that C does not contain **any** intervals.
   (b) Compute, with explanation, the length of the set C by using what you know about sequences to compute the lengths of the intervals that are thrown away.

2. We have defined the 'discrete derivative' of a sequence $a(k)$ by
   $$D_k [a(k)] = a(k+1) - a(k).$$
   In class we showed there is a 'Sum Rule' for discrete derivatives by showing that if $a(k)$ and $b(k)$ are sequences then
   $$D_k [(a+b)(k)] = D_k [a(k)] + D_k [b(k)].$$
   (a) Extend this type of argument by determining the 'Product Rule' for discrete derivatives. That is, if $a(k)$ and $b(k)$ are sequences with domain $k = 0, 1, 2, 3, \cdots$ then determine, with explanation, the right hand side of the following formula
   $$D_k [(ab)(k)] =$$
   [Hint: Read the proof in the textbook of the product rule for regular derivatives.]
   (b) Illustrate that your formula works by computing the discrete derivative $D_k [k^3] = 3k^2 + 3k + 1$ by using the product rule on $(ab)(k) = a(k)\,b(k)$ where $a(k) = k^2$ and $b(k) = k$.