

Due September 8, 2003

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Name

Directions: Be sure to follow the guidelines for writing up projects as specified in the course information sheet (passed out on the first day of class). Whenever appropriate, use in-line citations, including page numbers and people consulted when you present information obtained from discussion, a text, notes, or technology. **Only write on one side of each page.**

*“In mathematics you don’t understand things. You just get used to them.”* — John von Neumann

### Project Description

For this first project please submit your efforts on any two (2) of the following.

1. There are many ways to specify geometric objects to an audience. For example, when describing spheres it is usually easiest to give the center and radius since those two bits of information completely specify a unique sphere. By using the points  $A(-1, 5, 3)$  and  $B(6, 2, -2)$ , show that it is also possible to specify a sphere as the set of points  $P$  such that the distance from point  $P$  to a fixed point  $A$  is twice the distance from point  $P$  to another point  $B$ . In particular,
  - (a) Explain how you know the set of points is really a sphere.
  - (b) Give the center and radius of that sphere.
2. We already know that the special vectors  $\hat{\mathbf{i}} = \langle 1, 0 \rangle$  and  $\hat{\mathbf{j}} = \langle 0, 1 \rangle$  in  $R^2$  allow us to write any vector in  $R^2$  as the sum of scalar multiples of  $\hat{\mathbf{i}}$  and  $\hat{\mathbf{j}}$ . Specifically,  $\langle a, b \rangle = a\hat{\mathbf{i}} + b\hat{\mathbf{j}}$ . Now show that if  $\vec{v}$  and  $\vec{w}$  are any two **non-parallel, non-zero** vectors in  $R^2$  and  $\vec{u}$  is any vector in  $R^2$ , then there have to be scalars  $r, s$  for which  $\vec{u} = r\vec{v} + s\vec{w}$ . Specifically,
  - (a) Give a geometric explanation for this fact.
  - (b) Give an algebraic explanation using components.
3. This is more of a computational problem than the other two but still requires a full written explanation.

Use the dot product to

- (a) find the angle between a diagonal of a cube and an edge of the cube that shares a vertex with the diagonal.
- (b) find the angle between a diagonal of a cube and a diagonal of a face of the cube. Please use two diagonals that share a common vertex of the cube.