

November 10, 2000

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Name

Directions: Be sure to include in-line citations, including page numbers if appropriate, every time you use the results of discussion, a text, notes, or technology. **Only write on one side of each page.**

*"Personally, I'm always ready to learn, although I do not always like being taught."* – Winston Churchill

**Problems**

1. Let  $G$  be the group of rotational symmetries of a cube  $C$ . Two regular tetrahedra  $\Delta$  and  $\Delta'$  can be inscribed in  $C$ , each using half of the vertices. What is the order of the stabilizer of  $\Delta$ ?
2. Do **one** of the following.
  - (a) Prove the formula  $|G| = |Z(G)| + \sum |C|$  where the sum is over the conjugacy classes containing more than one element and  $Z(G)$  is the center of  $G$ .
  - (b) Rule out as many of the following as possible as Class Equations for a group of order 10.
    - i.  $1 + 1 + 1 + 2 + 5$
    - ii.  $1 + 2 + 2 + 5$
    - iii.  $1 + 2 + 3 + 4$
    - iv.  $1 + 1 + 2 + 2 + 2 + 2$
3. Let  $Z(G)$  be the center of a group  $G$ . Prove that if  $G/Z$  is a cyclic group, then  $G$  is abelian and hence  $G = Z(G)$ .